Global gyrokinetic particle simulation of toroidal Alfvén eigenmodes excited by antenna and fast ions

Wenlu Zhang,1,2,a) Ihor Holod,2 Zhihong Lin,2,3 and Yong Xiao2,4

1CAS Key Laboratory of Plasma Physics, University of Science and Technology of China, Hefei, Anhui 230026, China
2Department of Physics and Astronomy, University of California, Irvine, California 92697, USA
3Fusion Simulation Center, Peking University, Beijing 100871, China
4Institute for Fusion Theory and Simulation, Zhejiang University, HangZhou, Zhejiang 310058, China

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I. INTRODUCTION

Energetic particles can be created in magnetically confined plasmas through fusion reactions (\(\Phi\)-particles) and auxiliary heating such as neutral beam injection (NBI) or radio frequency (RF) heating. Generally, \(\Phi\)-particles distributions are isotropic in velocity space, while the NBI mainly contributes to passing populations and a large portion of RF heated particles are deeply trapped particles. These energetic particles are subject to the interaction with magnetohydrodynamic (MHD) instabilities, microturbulence, stochastic magnetic field, and classical collisional and orbital effects. The interaction between energetic particle and background field could be twofold: the microturbulence could affect the confinement of the energetic particles; on the other hand, they may drive a new type of instabilities, for example, Alfvén eigenmodes (AEs) and nonperturbative energetic particle modes (EPMs) etc. Fusion experimental and theoretical studies, especially recent large scale gyrokinetic particle simulations and NBI experiments on DIII-D show that transport of high-energy \(\Phi\)-particles by ion temperature gradient microturbulence driven by the thermal plasma is negligible in burning plasmas. However, the toroidal nature of tokamak produces gaps in the continuous spectrum of Alfvén waves, which are populated by discrete, undamped shear Alfvén gap modes, for example, the toroidicity-induced Alfvén eigenmode (TAE), the reversed shear Alfvén eigenmode (RSAE), and the \(\beta\)-induced Alfvén eigenmode (BAE). These modes could be readily destabilized by energetic particles in burning plasmas, which induce large cross-field transport of energetic particles through wave-particle interactions and, therefore, degrades the plasma confinement.

The global gyrokinetic toroidal code (GTC) has been successfully applied to simulate energetic particle transport by electrostatic turbulence and Alfvén eigenmodes, RSAE and BAE. In this article, we will further demonstrate and verify its electromagnetic capability through a series of global gyrokinetic simulations of AEs in cylindrical and toroidal geometries. Such global gyrokinetic simulations take advantage over the MHD model that they can capture the full kinetic physics, which are essential for retaining the important background damping of various types such as collisional damping, Landau damping, continuum damping, radiative damping, etc. Recently, a number of gyrokinetic global codes have been developed to simulate the low-\(n\) TAEs. For example, Mishchenko et al. use of particle code GYGLES, Lang et al. extended GEM code for the TAE simulations, and Lauber et al. developed a gyrokinetic eigenvalue code, LIGKA.

This paper is organized as follows. The gyrokinetic simulation modelnow described in Sec. II, the Alfvén wave simulation in cylindrical geometry is presented in Sec. III, the toroidal simulation results of TAE excitation by antenna and fast ions are presented in Sec. IV, and Sec. V summarizes this work.

II. GYROKINETIC SIMULATION MODEL

In this section, the electromagnetic gyrokinetic model used by GTC is described first. Then, the ideal MHD theory is recovered in the long wave-length limit. Next, the TAE dispersion relation is derived from the reduced equations. Finally, the TAE excitation by antenna is explained.

A. Formulation for gyrokinetic simulation

The gyrokinetic equation used to describe the plasma in toroidal systems is expressed in 5D phase space (gyrocenter position \(X\), magnetic moment \(\mu\), and parallel velocity \(v_\parallel\))
\[
\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \mathbf{E} \cdot \nabla - \mathbf{A} \cdot \nabla = 0,
\]

(1)

\[
\mathbf{v} = \frac{\mathbf{E}}{B_0} + \mathbf{v}_E + \mathbf{v}_c + \mathbf{v}_g,
\]

(2)

\[
\dot{\mathbf{v}} = -\frac{B^*}{m_e B_0} (\mu \nabla B_0 + Z_e \nabla \phi) - \frac{Z_e}{m_e} \frac{\partial A}{\partial t},
\]

(3)

where index \(x = e, i, f\) stands for particle species of electron, thermal ion, and fast ion, respectively, \(Z_e\) is the particle charge, \(m_e\) is the particle mass, \(\phi\) and \(A\) are the gyroaveraged perturbation of electrostatic and vector potentials, \(B_0 = B_0 + \delta B\) is the equilibrium magnetic field, \(B^* = B_0 + \delta \mathbf{B}\), \(B_0 = (B_0) / \Omega_z\), \(\nabla \times \mathbf{b}_0\), \(\delta \mathbf{B} = \nabla \times (\mathbf{A} \mathbf{b}_0)\). The \(\mathbf{E} \times \mathbf{B}\) drift \(\mathbf{v}_E\), the curvature drift \(\mathbf{v}_c\), and the grad-\(\mathbf{B}\) drift are given by

\[
\mathbf{v}_E = \frac{\mathbf{c}_0 \times \nabla \phi}{B_0},
\]

\[
\mathbf{v}_c = \frac{\nabla \times \mathbf{b}_0}{\Omega_z},
\]

\[
\mathbf{v}_g = \frac{\mu}{m_e \Omega_z} \mathbf{b}_0 \times \nabla B_0.
\]

In order to describe the properties of electrons for the mesoscale turbulence, GTC adopts the fluid-kinetic hybrid electron model, which consist of a dominant adiabatic part and a high-order kinetic response. The lowest order adiabatic part is used in this work to simulate the AEIs, which is obtained by integrating the electron gyrokinetic equation in the drift kinetic limit and gives the continuity equation of the electron density

\[
\frac{\partial n_e}{\partial t} + \mathbf{B}_0 \cdot \nabla \left( \frac{n_e \delta u_{el}}{B_0} \right) + \mathbf{B}_0 \cdot \nabla \left( n_{el} \right) - n_{e0} \left( \mathbf{v}_{ce} + \mathbf{v}_E \right) \cdot \nabla \frac{B_0}{B_0} = 0,
\]

(4)

where \(\mathbf{v}_{ce} = \mathbf{b}_0 \times \nabla \left( \delta P_{el} + \delta P_{pe}\right) / (n_{e0} m_e \Omega_z)\), \(\delta P_{el} = \int d\mathbf{v} m_e^2 \delta f_e, \delta P_{pe} = \int d\mathbf{v} m_i \delta f_i, \delta n_{e0} = \int d\mathbf{v} f_{e0}\). The electron parallel fluid velocity in the above equation can be calculated by using the parallel Ampère’s law

\[
n_{e0} e \delta u_{el|e} = \frac{c}{4\pi} \nabla^2 \delta \mathbf{A} + \sum_{x=i,f} n_{e0} Z_x \delta u_{el|x}.
\]

(5)

Here, the vector potential is obtained using the Faraday’s law

\[
\frac{\partial \delta \mathbf{A}}{\partial t} = \mathbf{v}_{ce|e} (\phi_{e0} - \phi) \equiv \mathbf{v}_{ce|e} \phi_{ind}.
\]

(6)

The effective potential \(\phi_{e0}\) is calculated by integrating the leading order terms of \(\phi_0 / (k_0^2 v_0^2)\) in the electron gyrokinetic equation assuming the uniform electron density,

\[
\frac{e \phi_{e0}}{T_e} = \frac{n_e}{n_{e0}}.
\]

(7)

This system is closed with the gyrokinetic Poisson’s equation

\[
\frac{Z_e n_i}{T_i} (\phi - \tilde{\phi}) = \sum_{x=i,e,f} Z_x \delta n_x,
\]

(8)

where \(\tilde{\phi}\) is the second gyrophase-averaged potential.

A more complete formulation with non-uniform thermal plasmas can be found in Ref. 27.

**B. Reduction to ideal MHD theory**

We now show that the gyrokinetic simulation model retains ideal MHD modes, for example, TAE, by reducing our equations in the long wavelength limit and no parallel electric field, \(\phi_{e0} = 0\). For uniform thermal plasmas, the linearized continuity equation (4) can be written

\[
\frac{\partial}{\partial t} \left( \frac{\delta n_e}{n_{e0}} \right) + \mathbf{B}_0 \cdot \nabla \left( \frac{\delta u_{el|e}}{B_0} \right) = 0.
\]

(9)

The Possion equation (8) becomes

\[
\frac{c^2}{4\pi e} \nabla \cdot \left( \frac{1}{\nu_A^2} \nabla \phi \right) = \delta n_e.
\]

(10)

where \(\nu_A = B_0 / \sqrt{4\pi n_e m_i}\) is the Alfvén velocity. Applying \(\nabla^2\) on Eq. (6) gives

\[
\frac{1}{c \frac{\partial}{\partial t}} \left( \nabla^2 \mathbf{A} \right) = -\nabla^2 \left( \mathbf{b}_0 \cdot \nabla \phi \right).
\]

(11)

The inverse Ampère’s law Eq. (5) reads

\[
\delta u_{el|e} = c \frac{e^2}{T_e} \frac{\nu_A^2}{\nu_D} \nabla^2 \left( \mathbf{b}_0 \cdot \nabla \phi \right).
\]

(12)

The ion density and ion parallel velocity have been ignored in Eqs. (10) and (12), since the ion parallel current is much smaller than the electron parallel current. Equations (9)–(12) can be combined into an eigenmode equation:

\[
\frac{\partial^2}{\partial t^2} \left\{ \nabla_\perp \cdot \left( \frac{1}{\nu_A^2} \nabla_\perp \phi \right) \right\} - \mathbf{B}_0 \cdot \nabla \left[ \frac{1}{B_0} \nabla^2_\perp \left( \mathbf{b}_0 \cdot \nabla \phi \right) \right] = 0.
\]

(13)

Finally, applying the Fourier transforms in time to the above equation, we arrive at

\[
\omega^2 \nabla_\perp \cdot \left( \frac{1}{\nu_A^2} \nabla_\perp \phi \right) = -\mathbf{B}_0 \cdot \nabla \left[ \frac{1}{B_0} \nabla^2_\perp \left( \mathbf{b}_0 \cdot \nabla \phi \right) \right],
\]

(14)

which recovers the ideal MHD equations.20

**C. TAE dispersion relation**

Now, we consider perturbations with high toroidal mode number \(n\) and use \(\delta \equiv \epsilon / n \sim O(k_i / k_\perp)\) with \(\epsilon = r / a\) as an expansion parameter to develop an asymptotic solution of Eq. (14). Employing the high-\(n\) ballooning mode representation, the perturbed quantities can be expressed as
where the coordinate system \((\Psi, \theta, \zeta)\) is adopted, the \(\theta\) domain of \(\phi\) extends from \(-\infty\) to \(\infty\), \(2\pi \Psi\) is the poloidal flux between the magnetic axis and a constant \(\Psi\) surface. The equilibrium magnetic field \(B_0 = \nabla x \times \nabla \Psi\), and \(x = z - q(\Psi)\theta\) is the magnetic field line label in a toroidal system, \(\theta\) and \(\zeta\) are the generalized poloidal and toroidal angles with a period of \(2\pi\), and \(q\) is the safety factor and is a function of \(\Psi\) only. In this coordinate system, \(B_0, \nabla \theta = (\nabla x \times \nabla \Psi) \cdot \nabla = J^{-1}, B_0, \nabla = J^{-1} \partial_\theta\), and \(J = q(\Psi) R^2 / (B_0^2 \partial_\theta)\) is the Jacobian. It is clear that Eq. (14) is satisfied by the perturbed quantities over an infinite range in \(\theta\) with no periodicity constrain. Then, we express \(\phi\) by the WKB representation,

\[
\vphitilde = \Phi(\Psi, \theta, \delta) \exp[i \varphi(x, \Psi)],
\]

where \(\varphi\) describes the rapid cross field variations and \(\Phi\) is the slow variations along the field lines on the equilibrium scale, so that \(B \cdot \nabla \varphi = 0\). For an axisymmetric toroidal system, \(\varphi\) is separable and can be expressed as \(\varphi = n(x + \int \theta_0(\Psi) d\Psi)\), where \(\theta_k\) is to be determined by a higher-order radial nonlocal analysis. In the lowest order in \(\delta\), Eq. (14) then reduce to a single second-order differential equations in \(\theta\) for every \(\Psi\) and \(\theta_k\). The final equation is given by

\[
J^{-1} \frac{\partial}{\partial \theta} \left[ J B^2 \frac{\partial}{\partial \theta} \Phi \right] + \frac{\partial^2}{\partial_\theta^2} |\nabla x|^2 \Phi = 0. \tag{15}
\]

For the purpose of analytical studies, we consider an axisymmetric, large aspect ratio, low-\(\beta\) toroidal plasma with concentric, circular magnetic surfaces, which is used as the analytic equilibrium geometry. The generalized radius, poloidal, and toroidal angles are related to the geometrical radius and angles \((\bar{r}, \bar{\theta}, \bar{\zeta})\) through \(\bar{r} = r, \bar{\theta} = \theta + \epsilon \sin \theta, \text{and } \bar{\zeta} = \zeta\). Thus, the lab coordinate can be expressed as \(x = (R_0 + \bar{r} \cos \bar{\theta}) \cos \bar{\zeta}, y = -(R_0 + \bar{r} \cos \bar{\theta}) \sin \bar{\zeta}, z = \bar{r} \sin \bar{\theta}\). Therefore, we have \(|\nabla x|^2 = (q(\theta))^2 \sin^2 \bar{\theta} = (1 + s^2 \theta^2) / \eta^2 (1 + c \cos \theta)^2\), where magnetic shear \(s = q r (1 + \epsilon \cos \theta) / q\). Then, the above equation can be rewritten as

\[
\frac{\partial}{\partial \theta} \left[ \frac{1 + s^2 \theta^2}{(1 + \epsilon \cos \theta)^2} \frac{\partial}{\partial \theta} \Phi \right] + \Omega^2 (1 + 4 \epsilon \cos \theta) \frac{(1 + s^2 \theta^2)}{(1 + \epsilon \cos \theta)^2} \Phi = 0.
\]

If we prescribe \(A = (1 + s^2 \theta^2) / (1 + \epsilon \cos \theta)^2\) and \(\Phi = \Phi^{-1/2} \phi\), the high-\(n\) ballooning equation, Eq. (15), then reduces to

\[
\frac{\partial^2 \Phi}{\partial \theta^2} + \Omega^2 (1 + 4 \epsilon \cos \theta) \Phi + \left( \frac{\partial^2 A / \partial \theta^2}{4 A^2} \right) \Phi = 0, \tag{16}
\]

where \(\Omega = \omega / \omega_A, \omega_A = \nu_A^{(0)} / q R_0, \nu_A^{(0)} = \nu_A |_{\omega = \nu}\). The last term is the origin for the TAE eigenmode. Without this term, Eq. (16) is the standard Mathieu’s equation, which holds a frequency gap in the Alfvén continuum and the gap width is proportional to \(2 \epsilon\).

### D. Basic theory of TAE excitation by antenna

In Sec. II C, the dispersion relation for TAE can be obtained from Eq. (16), which has been verified by TAE simulations excited by initial perturbations. Now, we are going to discuss the eigenmode excitation by antenna, which provides a way to accurately determine the eigenfrequency, damping rate, and mode structure for the damped modes in the linear and nonlinear initial value code. In GTC, the antenna is implemented through an extra synthetic potential \(\phi_{\text{ant}}\) added to induce potential \(\phi_{\text{ind}}\)

\[
\phi_{\text{ind}} = \phi_{\text{eff}} - \phi + \phi_{\text{ant}}
\]

then Eq. (14) becomes

\[
\frac{\partial^2}{\partial \theta^2} \left[ \nabla \cdot \left( \frac{1}{R^2_A} \nabla \phi \right) \right] - B_0 \cdot \nabla \left[ \frac{1}{R^2_A} \nabla \phi (\phi_{\text{ant}}) \right] = 0. \tag{17}
\]

Suppose that a standing sinusoidal signal is loaded on the antenna at time \(t = 0\),

\[
\phi_{\text{ant}} = \phi_{\text{ant}}(\Psi, \theta, \zeta) \cos(\omega_{\text{ant}} t),
\]

and potential \(\phi\) can be separated into spatial and temporal components \(\phi = \phi_l(\Psi, \theta, \zeta) \phi_t(t)\). Here, the antenna drive frequency \(\omega_{\text{ant}}\) is chosen close to an particular eigenfrequency \(\omega_E\) of Eq. (17). Then, applying the Laplace transform to the above equation, we have

\[
\Phi_s = \frac{p \phi_s(0) + \phi_t(0)}{(p^2 + \omega_E^2)}
\]

\[
+ \frac{\omega_{\text{ant}}}{(p^2 + \omega_{\text{ant}}^2)(p^2 + \omega_E^2)},
\]

where \(\Phi_s\) is the Laplace transform of \(\phi_s\), \(S(\Psi, \theta, \zeta)\) represents the spacial dependence. The inverse Laplace transform gives

\[
\phi_t = \left\{ \begin{array}{ll}
\phi_t(+0) \cos(\omega_E t) + \omega_E^{-1} \phi_t(+0) \sin(\omega_E t) + \frac{S(\Psi, \theta, \zeta)}{2 \omega_E^2} \left( \sin(\omega_E t) - \omega_E t / \omega_E \right) & \omega_{\text{ant}} = \omega_E \\
\phi_t(+0) \cos(\omega_E t) + \omega_E^{-1} \phi_t(+0) \sin(\omega_E t) + \frac{S(\Psi, \theta, \zeta)}{\omega_E^2 - \omega_{\text{ant}}^2} \left( \sin(\omega_{\text{ant}} t) - \omega_{\text{ant}} \sin(\omega_{\text{ant}} t) \right) & \omega_{\text{ant}} \neq \omega_E.
\end{array} \right.
\]

\[\tag{18}\]
In the first scenario, the antenna drive frequency equals to the eigenfrequency, which corresponds to a linearly growing mode with respect to time, with a frequency $\omega_{E}$. However, in the second scenario, the antenna frequency is different from the eigenfrequency, and the corresponding mode is purely oscillating waves, which contains both the drive frequency $\omega_{\text{ant}}$ and the eigen-frequency $\omega_{E}$.

### III. ALFVÈN EIGENMODE SIMULATIONS IN CYLINDRICAL GEOMETRY

The electromagnetic capability is first demonstrated by shear Alfven continuum spectrum in Fig. 1, where a initial perturbation is applied to the adiabatic electron density $\delta n_e$ with a given mode number $m/n=8/5$. We uses a screw pinch in this simulation with a linear safety factor $q = 1.0 + r/a$ and a constant magnetic shear $s = 0.2$ in the cylindrical geometry. The stars in Fig. 1 are the Alfven frequency measured at different radial locations in the simulation, while the solid line is the theoretical prediction $\omega = |k||v_A| \propto |(n - m/q)|$, where the vertical axis is normalized with the local Alfven frequency $\omega_A$ at $r = 0.5a$.

### IV. TAE EXCITATIONS BY ANTENNA

The electromagnetic capability is further demonstrated in toroidal simulation by TAE excitations using an external synthetic antenna. The eigenmode frequency, mode structure, and TAE gap size can be precisely measured in our simulation.

To recover the basic fluid properties of TAE, kinetic effects of the background thermal ions and electrons have been suppressed in the simulations in this section.

For low toroidal mode number $n$, the TAE mode frequency is determined by the breaking of shear Alfven continuum spectra. Given a toroidal number $n$, the coupling between adjacent poloidal harmonics of Alfven waves causes a frequency gap in the continuous spectra. The radial position $r$ corresponding to a frequency gap is determined by the safety factor profile and toroidal/poloidal mode numbers. In our simulations, the inverse aspect ratio is 0.15 at $r = 0.5a$, safety factor has a linear profile $q = 1.0 + r/a$, and the toroidal mode number is chosen as $n = 5$. At $r = 0.5a$, $m = 7$ and $m = 8$ modes have the same $|k||v_A| = |n-m/q|/R$ and form the local TAE gap. The TAE eigenfrequency can be determined by the spectrum of vector potential, which is dominated by the antenna drive frequency, the upper accumulating frequency, the lower accumulating frequency, and the TAE frequency. The TAE frequency moves from right above the lower accumulating point up to the middle of the gap when magnetic shear increases. Therefore, depending on the magnetic shear, the TAE frequency and the lower accumulating point may not be well separated. In our simulation, the magnetic shear is $s = 0.2$, so the TAE frequency and the lower accumulating frequency is too close to be distinguished.

The TAE is first excited by an antenna with a frequency right in the middle of the TAE gap, $\omega_{\text{ant}} = \omega_A = v_A/(2qR)$, see Fig. 2. The upper panel is the spectrum of the antenna, the middle is the spectrum of the response vector potential $A_{||}$, and the lower is the time history of $A_{||}$. In the response vector potential, two dominating frequencies other than the antenna frequency are the upper accumulating frequency, and the TAE frequency (lower accumulating point), respectively. This is held naturally by Eq. (11) for an ideal MHD case, where $E_{||} = 0$ and $A_{||} = \nabla_{||} \phi \propto \partial_{||} \phi$ for a standing wave.

Next, the antenna’s frequency is set to the TAE frequency, $\omega_{\text{ant}} = \omega_{\text{TAE}}$. In this scenario, Fig. 3, the only dominate frequency in the response $A_{||}$ is the TAE eigenfrequency, and as predicted by the analytic theory, the time history of vector potential $A_{||}$ grows linearly with time because of the
weak damping. As shown in Eq. (18), the width of the peaks in Fig. 3 depends on the eigen-frequencies, the driving frequency, and the number of oscillations used to calculate the mode frequency in the simulation. If damping mechanism is included in the simulation, the damping rate of the system affects the peak width as well. The mode structure is shown in Fig. 4, where the upper panel is that of the vector potential \( A_\| \) and the lower is the electrostatic potential \( \phi \). It is evident that the electrostatic potential shows a ballooning structure, while the vector potential shows an anti-ballooning structure.

The dominant upper accumulating frequency and TAE eigenfrequency in Fig. 2 can be used to estimate the TAE gap size. Fig. 5 shows the gap size dependence on the local aspect ratio \( \epsilon \), where the stars are simulation results for \( \epsilon = 0.05, 0.1, 0.15, 0.2 \), respectively. The frequency has been normalized using the Alfvén frequency \( \omega_{\text{AE}} = v_A/(2qR) \). The upper and lower frequencies are fitted with solid and dotted lines, respectively, which shows a clear linear dependence to aspect ratio: \( \omega / \omega_{\text{TAE}} = 1 \pm 1.35 \epsilon \). This gap size is slightly bigger than the theory predictions in Sec. II, where we employed a simplified model and the third term in Eq. (16) is committed.

V. TAE EXCITATIONS BY FAST IONS

The MHD capability of gyrokinetic particle simulation is further established by the TAE excitations using fast ions.

In this section, the aspect ratio is chosen to be \( a/R_0 = 0.3 \) (\( a \) and \( R_0 \) are the minor and major radius of tokamak, respectively). The fast ions are loaded with a Maxwellian distribution that holds the following on-axis parameters:

- \( v_f = v_A, \eta_f / n_0 = 0.07, a = 80 \rho_f, k_0 \rho_f = 0.2, \beta_e = 4\pi n_0 T_e / B_0^2 = 0.125, T_f = 16 T_e, \) where \( v_f \) is the thermal velocity of
the fast ions. The density gradient of fast ions peak at \( r = 0.5a \) and \( q = 1.5 \) surface such that two adjacent harmonics, \( m = 7 \) and \( m = 8 \), dominate the \( n = 5 \) mode. In the following part, the \( m = 8 \) TAE harmonics of the \( n = 5 \) mode is used as an example to investigate the linear TAE properties excited by fast ions.

As shown in Fig. 6, the fast ion excited TAE mode grows exponentially, with a growth rate of \( \gamma / \omega_{TAE} \approx 6\% \), while the theory prediction\(^{12} \) gives a growth rate of \( \gamma / \omega_A = 8\% \). The imaginary part of the mode is \( \pi / 2 \) leading the real part in phase, which means that this wave is a traveling wave and propagates in the fast ion diamagnetic direction. The mode frequency is slightly lower than the antenna result in Sec. III due to the nonperturbative kinetic contributions from the fast ions. The mode structure, Figs. 7 and 8, of TAE excited by fast ions shows a clear radial symmetry breaking when comparing with that excited by antenna, which is due to the nonperturbative kinetic effects of fast ions. The radial structure of the ballooning mode is determined by the radial eigenmode equation. In the local 1D theory, the ballooning mode has no radial structure (i.e., \( k_r = 0 \) at a poloidal angle \( \theta = \theta_f \)) because of the radial symmetry (Fig. 4). Any breaking of the radial symmetry, i.e., radial variations of pressure gradient and \( E_r \), etc., leads to the radial dependence of \( \theta_k \) (ballooning angle). Then, the radial mode structure can be twisted (Fig. 7) and a 2D global eigenmode theory\(^{28} \) is needed to solve the radial mode structure.

VI. SUMMARY

In summary, the MHD capability of gyrokinetic particle simulation has been verified through the simulations for the shear Alfvén eigenmodes in cylindrical and tokamak geometries by the GTC. A synthetic antenna is implemented in GTC to provide a way to precisely measure the eigenmode frequency, damping rate and mode structures. The Alfvén eigenmode simulation in cylindrical geometry is verified, where the measured shear Alfvén eigen-frequency dependence on the radial position agrees with the theoretical predictions. The antenna excitation provides the verifications of the TAE mode frequency, gap width, and mode structure. The measured gap size shows a linear dependence on the local aspect ratio. The TAE mode structure excited by fast ions shows a significant radial symmetry breaking relative to the...
antenna excitation due to the nonperturbative contributions from the fast ions. Through these verifications, we demonstrated that the shear Alfvén modes can be treated accurately with the gyrokinetic PIC method.

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