Transport of Energetic Particles by Microturbulence in Magnetized Plasmas

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Transport of energetic particles by the microturbulence in magnetized plasmas is studied in gyrokinetic simulations of the ion temperature gradient turbulence. The probability density function of the ion radial excursion is found to be very close to a Gaussian, indicating a diffusive transport process. The particle diffusivity can thus be calculated from a random walk model. The diffusivity is found to decrease drastically for high energy particles due to the averaging effects of the large gyroradius and orbit width, and the fast decorrelation of the energetic particles with the waves.

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Energetic particles can be generated in magnetically confined plasmas by fusion reactions and auxiliary heating. They can be subjected to the diffusion by macroinstabilities [1], microturbulences [2], a stochastic magnetic field [3], and classical collisional and orbital effects [4]. The confinement of the energetic particles is a critical issue in the International Thermonuclear Experimental Reactor (ITER) [5], since the ignition relies on self-heating by the energetic fusion products. The diffusion of the energetic particles such as the cosmic rays by microscopic turbulence is also an important scientific issue in the space and astrophysical plasmas [6]. Earlier fusion experimental studies [4] with a higher born energy and the newer experiments [4] with a lower born energy showed some evidence of the correlation between the excitation of the microturbulence and the redistribution of energetic ions produced by the neutral beam injection (NBI). Some recent theoretical [10] and computational [11] studies also suggested a significant transport level of the energetic particles driven by the microturbulence.

To resolve this discrepancy, here we study the diffusion of the energetic particles by the microscopic ion temperature gradient (ITG) turbulence in large scale first-principles simulations of fusion plasmas using the global gyrokinetic toroidal code (GTC) [12]. The ion radial spread as a function of energy and pitch angle is measured in the steady-state ITG turbulence. The probability density function of the radial excursion is found to be very close to a Gaussian, indicating a diffusive transport from a random walk process. The radial diffusivity as a function of the energy and pitch angle can thus be calculated using the random walk model. We find that the diffusivity decreases drastically for high energy particles due to the averaging effects of the large gyroradius and banana width, and the fast decorrelation of the energetic particles with the ITG oscillations. By performing the integration in phase space, we can calculate the diffusivity for any distribution function. The NBI ion diffusivity driven by the ITG turbulence is found to decrease rapidly for the born energy up to an order of magnitude of the plasma temperature and more gradually to a very low level for higher born energy. This result may explain the differences between the older experiments [4] with a higher born energy and the newer experiment [9] with a lower born energy (relative to the plasma temperature).

Fully self-consistent ITG turbulence simulation.— This study employed a well bench-marked, massively parallelfull torus gyrokinetic toroidal code (GTC) and used representative parameters of tokamak H-mode core plasmas which have a peak temperature gradient of thermal ions at a radial position \( r = 0.5a \) with the following local parameters: \( R_0/L_T = 6.9 \), \( R_0/L_n = 2.2 \), \( q = 1.4 \), \( \hat{s} \equiv (r/q)(dq/dr) = 0.78 \), \( T_e/T_i = 1 \), and \( \epsilon = a/R_0 = 0.36 \). Here \( R_0 \) is the major radius, \( a \) is the minor radius, \( L_T \) and \( L_n \) are the temperature and density gradient scale lengths, respectively, \( T_e \) and \( T_i \) are the ion and electron temperature, and \( q = 0.854 + 2.184(r/a)^2 \) is the safety factor. In the full torus nonlinear simulation of a \( a = 500r_i \) tokamak with \( \rho_i \) measured at \( r = 0.5a \), we calculated 800 transit times of \( 4 \times 10^8 \) bulk marker particles (guiding centers), and interactions of these particles with self-consistent electrostatic potential represented on \( 4 \times 10^7 \) spatial grid points to address realistic reactor-grade plasma parameters covering disparate spatial and temporal scales. A more complete simulation model is described in Ref. [13]. The simulation starts with very small random fluctuations which grow exponentially due to the toroidal ITG instability as evident in the early part of the time history of the ion heat conductivity shown in the lower panel of Fig. 1. Zonal flows are then generated through modulational instability [14,15] and the ITG instabilities are saturated at the time of \( t = 250L_T/v_i \) through random shearing by the zonal flows [16]. Finally, the nonlinear coupling of ITG-zonal flows leads to a fully developed turbulence after...
For each velocity grid point, we initiate 50 000 particles uniformly distributed in the radial domain of \( r/a \in [0.45, 0.55] \) where the intensity of the turbulence is maximal. We calculate the mean-squared radial displacement of each group of particles \( \langle \Delta r^2 \rangle = \sum_{i=1}^{N} N^{-1} \sum_{t=1}^{T} (r_i(t) - r_i(0))^2 \), where \( r_i(0) \) and \( r_i(t) \) are the radial position of the \( i \)th particle at time \( t = 0 \) and time \( t \), respectively. The time history of the radial displacements for several energy groups (averaged over the pitch angle) are shown as the solid lines in the upper panel of Fig. 1. To isolate the effects of turbulence scattering, we also calculate the radial displacements of the same ions in another simulation where the ITG turbulence is suppressed. These displacements of the equilibrium orbits are plotted as dashed lines in the same figure. After a few bounce times \( \tau = 2\pi qR/\sqrt{eN} \) in the equilibrium displacement subtracted by the equilibrium displacement increases linearly with time for all energy groups in the steady-state ITG turbulence. This feature indicates that the radial excursion of the ions due to the turbulence scattering is a diffusive process.

The diffusive process is confirmed by the probability density function (PDF) of the radial displacement for each energy group. The PDF obtained at \( t = 800L_T/v_i \) is indeed very close to a Gaussian as shown in the upper panel of Fig. 2. Here, the skewness is \( S = \sqrt{N} \sum_{i=1}^{N} (x_i - \bar{x})^3/\sigma^3 \) and the kurtosis \( K = N \sum_{i=1}^{N} (x_i - \bar{x})^4/\sigma^4 \). Consistent with the trend of the mean-squared displacements in Fig. 1, the standard deviation of the PDF in Fig. 2 decreases with increasing energy after it peaks at an energy of \( E = 2T_e \). This peak energy is found to be consistent with the toroidal resonance condition for the ITG modes. That is, the drift resonance condition of \( \omega = \omega_d \) is satisfied at \( E = 2T_e \) when averaging over the pitch angle. Here the drift frequency is \( \omega_d = k_g v_d \), \( v_d = (v^2 + v_p^2)/(2R) \), and the ITG frequency is \( \omega = k_g v_{ph} \), where \( \Omega \) is the ion cyclotron frequency and \( k_g = nq/r \). The linear phase velocity \( v_{ph} \) is measured in the simulation and is roughly a constant [13] for the modes \( k_g p_i = [0, 0.3] \), which have significant amplitudes in the nonlinear state.

The diffusive nature of the ITG turbulent transport is further supported by the fact that the radial profile of the...
the radial excursion \( r/a \) for particle energy \( E/T_e = 1 \) (black), 2 (blue), 4 (green), 16 (orange). \( \sigma \), \( \beta \), and \( K \) are the standard deviation (in unit of \( \rho_i \)), skewness, and kurtosis, respectively.

Transport of energetic particles.—Since the radial excursion of the ions is diffusive, a phase-space-resolved diffusivity can be defined using the random walk model

\[
D(E, \xi) = \frac{\Delta \sigma^2}{(2 \Delta t)},
\]

where \( \Delta \sigma^2 \) is the change of the PDF standard deviation of the net radial displacement for each group of ions with energy \( E \) and pitch angle \( \xi \) during a time interval of \( \Delta t \) between \( t = 400L_T/v_i \) and \( t = 800L_T/v_i \) (when the turbulence is in a steady state). This radial diffusivity as a function of \( (E, \xi) \) is plotted in Fig. 3. The diffusivity is relatively smooth across the pitch angle \( \xi \) with no sharp resonance in the entire phase space. This is consistent with the diffusive process and the transport could therefore be described by a quasilinear theory [19]. As a consistency check, we calculate the diffusivity of the thermal ions by averaging the diffusivity \( D \) over a Maxwellian distribution function with a temperature of \( T_e, D_0 = \int DF_m d^3v \). We find that this diffusivity \( D_0 \) based on the random walk model is very close to an effective particle diffusivity \( D_i \) of thermal ions measured in the simulation, \( D_0 = 1.1D_i \). Here \( D_i = 2\chi_i/3 \) and \( \chi_i \) is 3.1\( \chi_{GB} \) calculated from the self-consistent heat flux using \( \chi_i = Q_i/(dT/\rho) \). \( Q_i = \int \frac{1}{2} \nu^2 \delta v \delta f d^3v \) measured in the simulation, where \( \nu \) is particle velocity, \( \delta f \) is the perturbed distribution function, and \( \delta v_r \) is the radial component of gyrophase-averaged \( \mathbf{E} \times \mathbf{B} \) drift. Note that the particles used to measure the radial diffusion are part of the plasma with both drag and scattering effects. However, the scattering effect dominates the radial diffusion (similar to test particles) because of the constraints of the quasineutrality and adiabatic electrons.

The diffusivity shown in Fig. 3 peaks at the resonant energy of \( E = 2T_e \) and decreases drastically for higher energy particles. This is due to the averaging effects of the large gyroradius and orbital width, and the fast decorrelation of the energetic particles with the waves for the high energy particles. To understand the physical mechanisms of the reduction of the diffusivity for energetic particle, we examine the scaling of the diffusivity with respect to the particle energy. Taking a cut of \( \xi = 0 \) in Fig. 3, we find that the diffusivity \( D \propto 1/E^2 \) for trapped energetic particles with \( E > 3T_e \). This scaling can be understood from the quasilinear theory of the diffusivity for trapped particles in the high energy limit,

\[
D \propto \sum_n \frac{e^2}{B^2} \frac{\rho_b}{|k_n|} J_n^2(k_n \rho_b) \int dE (k_n \rho_b) \delta (\omega - \omega_d + \rho_n \omega_b).
\]

Here the first Bessel function comes from the gyroaveraging and the second from the averaging over banana orbits, \( \omega_b = \sqrt{e}\nu_E/(2\pi qR) \) is the bounce frequency, \( e = r/R \) is the inverse aspect ratio, \( \rho_b \) is the banana width, \( \rho_n \) and \( \nu_E \) are the gyroradius and velocity of energetic particles, respectively, \( k_r \) and \( k_\theta \) are the wave number in the \( r \) and \( \theta \) direction, \( B \) is the magnetic field, \( \phi_a \) is the electrostatic potential, and \( p \) is the harmonics number. These two

FIG. 3 (color). Diffusivity \( D/D_i \) as a function of particle energy \( E/T_e \) and pitch angle \( \xi \).
averages give rise to a dependence of $D$ on $1/E$ when taking a large argument expansion of the Bessel function. Regarding the resonance condition, since $\omega_d$, $\omega_p \gg \omega$ for energetic particles, we need $p > 0$. Therefore the resonance condition becomes $\omega_d = p\omega_b$, or equivalently, $n\omega_{\text{pre}} = p\omega_b$, where $\omega_{\text{pre}} = qE/(mRr\Omega)$ is the precession frequency. This is the so-called drift-bounce resonance [20] underlying the ripple loss process, which gives rise to another dependence of $D$ on $1/E$ when integrating over $n$ since $\omega_{\text{pre}}$ is proportional to $E$. Hence, for energetic trapped particles, the diffusivity $D \propto 1/E^2$ accounting both the orbit averaging and the decorrelation process. For passing particles, the resonance condition of $\omega = E_b V_b$ and the orbital averaging will each give rise to an $E^{-1/2}$ dependence of $D$, so we expect that a diffusivity $D \propto 1/E$ for energetic passing particle. Indeed, when taking a cut of Fig. 3 at $\xi = 1$ and $-1$, we find that the diffusivity $D$ depends roughly on $1/E$ for $E > 20T_e$. These different scalings result in a larger diffusivity for the energetic passing particles than the energetic trapped particles as shown in Fig. 3.

The measured diffusivity in phase space as shown in Fig. 3 provides all information for calculating the diffusivity for arbitrary distribution function of energetic ions by taking integration over the energy and pitch angle. Of particular interest is the diffusivity for the NBI ions since most existing fusion experiments use the NBI heating. A steady-state slowing-down distribution function [21] is commonly used to describe the energetic NBI ions in a background plasma consisting of thermal ions and electrons. By solving the Fokker-Planck equation including a source term in plasma consisting of thermal ions and electrons. By solving the Fokker-Planck equation including a source term in plasma consisting of thermal ions and electrons.

FIG. 4. Diffusivity for a slowing-down distribution as a function of the NBI beam ion born energy ($E_b/T_e$) for passing (solid and dash) and trapped (dotted) particles.

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