Gyrokinetic simulations of nonlinear interactions between magnetic islands and microturbulence

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Abstract
A conservative scheme of kinetic electrons for gyrokinetic simulations in the presence of magnetic islands has been implemented and verified in the gyrokinetic toroidal code, where zonal and non-zonal components of all perturbed quantities are solved together. Using this new conservative scheme, linear simulation of kinetic ballooning mode has been successfully benchmarked with the electromagnetic hybrid model. Simulations of nonlinear interactions between magnetic islands and the ion temperature gradient (ITG) mode in a tokamak show that the islands rotate at the electron diamagnetic drift velocity. The linear ITG structure shifts from the island O-point toward the X-point due to the pressure flattening effect inside the islands, and the nonlinear ITG structure peaks along the magnetic island separatrix because of the increased pressure gradient there.

Keywords: gyrokinetic simulations, magnetic islands, microturbulence

(Some figures may appear in colour only in the online journal)

1. Introduction
Nonlinear interactions between macroscopic magnetic islands and microturbulence are important for the stability and transport in magnetic confinement plasmas. On one hand, magnetic islands generated by neoclassical tearing mode (NTM) [1, 2] or resonant magnetic perturbations [3, 4] can modify plasma profiles, flows within and around the islands, therefore influence turbulent transport significantly. Recent experiments [5, 6] on DIII-D tokamak and LHD stellarator have shown considerable reduction of cross-field thermal diffusivity at the O-point of magnetic island. A global gyrokinetic simulation [7] found that the reduction of microturbulence is related to the island flattening effect and the strong shear flows formed around the islands. On the other hand, microturbulence regulates plasma current and electron heat conductivity along and across the magnetic islands, thus affecting the island dynamics. An experimental observation of turbulence triggered fast recovery of NTM has recently been reported in the DIII-D tokamak [8]. Previous fluid simulations [9, 10] found that broad-spectral microturbulence can transfer energy to low-n modes and thus induce seed island [9]. Finally, turbulence-driven current can play an important role in the determination of NTM onset [10].

Global kinetic simulation of the nonlinear interactions between magnetic islands and microturbulence can be difficult. Firstly, electron density and flow calculated from particles cannot accurately satisfy electron continuity equation due to particle noises. This inconsistency leads to an incomplete cancellation of electrostatic and inductive electric fields, and gives rise to an unphysical component in parallel electric field [11, 12]. Secondly, it is difficult to analytically calculate the adiabatic response of electron in the presence of magnetic islands, where the zonal and non-zonal components of perturbed electromagnetic fields need to be defined with respect to the helical magnetic flux surface of the islands.
In this paper, we extend the conservative scheme of kinetic electrons in gyrokinetic simulation [11, 12] to incorporate magnetic islands and verify this new method in the gyrokinetic toroidal code (GTC) [13–15]. Our conservative scheme uses electron continuity equation and the Ampere’s law to evolve electron density and parallel flow, thus enforcing the consistency in the calculation of parallel electric field. The zonal and non-zonal components of all perturbed quantities are solved together, as required in the presence of magnetic island. By default, our model uses gyrokinetic ions and drift-kinetic electrons to incorporate kinetic effects, and the model can also be reduced to a fluid electron model if desirable. As a first step towards simulating the nonlinear interactions between NTM and microturbulence, we verify our model for drift-Alfvénic waves with kinetic electrons, and then use it to study the interactions between magnetic islands and microturbulence. In particular, simulation results of the kinetic Alfvén wave (KAW) in a slab-geometry agree well with the theoretical dispersion relation. We also demonstrate superior numerical properties of our conservative scheme when simulation grid size is larger than electron skin depth. Secondly, we benchmark the conservative model with the electromagnetic hybrid model [14] for linear kinetic ballooning mode (KBM) [16]. In the fluid electrons simulations, we find that the two models get very good agreement for the KBM growth rates and real frequencies. For the kinetic electrons simulations, the two models also get reasonable agreement. Finally, we use the conservative model with fluid electrons to study the interactions between a magnetic island and the ion temperature gradient mode (ITG) [17, 18]. In our simulations, we find that the magnetic island rotates at the electron diamagnetic drift velocity, which is consistent with the theory of the drift-tearing mode [19, 20]. We also observe that the magnetic island can significantly modify the ITG mode structures. In the linear simulation, the ITG structure shifts from the island O-point toward the island X-point due to the pressure flattening inside the island. In the nonlinear simulation, the ITG structure peaks along the magnetic island separatrix because of the increased pressure gradient there. Our results are in qualitative agreement with previous theoretical and numerical studies of the magnetic island effects on the ITG mode [21–25].

The remainder of this paper is organized as follows: section 2 describes the new conservative model, the verification of conservative model is given in the appendix. In the section 3, we use the model to study the interactions between the magnetic island and ITG turbulence. Section 4 gives a brief conclusion.

2. Gyrokinetic simulation model

2.1. Nonlinear gyrokinetic equations

GTC uses the gyrokinetic equation to study low frequency plasma waves, the nonlinear gyrokinetic equation [26, 27] is:

$$\frac{\partial}{\partial t} f(R, v_l, \mu, t) = \left( \frac{\partial}{\partial t} + \dot{R} \cdot \nabla + v_l \frac{\partial}{\partial v_l} - C \right) f = 0. \quad (1)$$

The distribution function $f(R, v_l, \mu, t)$ for ion or electron is described by five dimensional independent variables: the gyrocenter position $R$, the parallel velocity $v_l$, and the magnetic momentum $\mu$. The gyrocenter velocity is denoted by $\dot{R}$, the parallel acceleration is $v_l$, and the collision operator is $C$. We do not take any collision effects into account in this work and simply set $C = 0$. The equations for $\dot{R}$ and $v_l$ are:

$$\dot{R} = v_l \left[ \frac{B_0}{B_0} \frac{\delta B}{B_0} + \frac{c_b \times \nabla \delta \phi}{B_0} \right] + \frac{v_l^2}{\Omega} \nabla \times B_0 + \frac{\mu}{m \Omega} B_0 \times \nabla B_0, \quad (2)$$

$$v_l = -\frac{1}{m} \frac{B_0^*}{B_0} \cdot \left( \mu \nabla B_0 + Z \nabla \delta \phi \right) - \frac{Z}{mc} \frac{\partial \delta A_i}{\partial t}, \quad (3)$$

where $B_0$, $\delta B$, $\delta \phi$, $\delta A_i$ are background and perturbed magnetic field, perturbed electrostatic potential and parallel magnetic potential, respectively; $B_0 = B_0^* + B_{\Omega0}^* \nabla \times B_0$ and $\delta B = \nabla \times (\delta A_i B_0)$; $m, Z, c, \Omega = \frac{Z B_0}{mc}$ denote particle mass, electric charge, light speed and cyclotron frequency, respectively; $v_k, v_c, v_d \equiv E \times B$ drift velocity, magnetic curvature drift velocity, and magnetic gradient drift velocity, respectively.

In the $\delta f$ method [28, 29], both the distribution function $f$ and the propagator $\frac{d}{dt} = L_0 + \delta L$ and $f = f_0 + \delta f$. Correspondingly, the equation (1) can be expressed as $\frac{d}{dt} f = L f + \delta L$ or $\frac{d}{dt} f = (L_0 + \delta L)(f_0 + \delta f) = 0$. Where:

$$L_0 = \frac{\partial}{\partial t} + \left( v_l \frac{\partial}{\partial v_l} + \frac{v_l^2}{\Omega} \nabla \times B_0 \right) \cdot \nabla - \frac{1}{m} \frac{B_0^*}{B_0} \cdot \left( \mu \nabla B_0 \right) \frac{\partial}{\partial v_l}, \quad (4)$$

$$\delta L = \left( v_l \frac{\partial}{\partial v_l} + \frac{c_b \times \nabla \delta \phi}{B_0} \right) \cdot \nabla - \frac{1}{m} \frac{B_0^*}{B_0} \cdot Z \nabla \delta \phi \frac{\partial}{\partial v_l} - \frac{1}{m} \frac{\partial B_0}{B_0} \cdot \mu \nabla B_0 \frac{\partial}{\partial v_l} - \frac{Z}{mc} \frac{\partial \delta A_i}{\partial t} \frac{\partial}{\partial v_l}. \quad (5)$$

The equilibrium is defined by $L_0 f_0 = 0$, and can be approximated by a local Maxwellian for calculating perturbed distribution function:

$$f_0 = \frac{n_0}{(2\pi m v_{th0}^2)^{3/2}} \exp \left( -\frac{E}{T_0} \right). \quad (6)$$

Here $v_{th} = \sqrt{T_0/m}$, $E = \frac{1}{2} m v_{th}^2 + \mu B_0$ are thermal velocity and kinetic energy, $n_0$, $T_0$ are equilibrium density and temperature, respectively. Defining a weight variable as $w = \delta f/f$, the dynamic equation for $w$ can be expressed as:
In the particle-in-cell method, the electrons continuity equation and the Ampere equation accurately. As a result, the inductive and electric fields do not cancel with each other, leading to an unphysical component in the parallel electric field \( \nabla \mathbf{E}_p \). To close the system, electrostatic potential and parallel magnetic potentials are solved by the gyrokinetic Poisson equation [26] and the parallel Ampere’s law.

\[
\frac{Z_e^2 n_0}{T_0} (\delta \phi - \delta \phi) = Z_i \delta n_i + Z_e \delta n_e
\]

(8)

\[
\nabla^2 \delta A_i = -\frac{4\pi}{c} (Z_e n_e \delta u_{i|e} + Z_i \delta n_i)
\]

(9)

Equations (1)–(9) form a closed system for gyrokinetic simulation. When the wavelength is longer than electron gyroradius, the above gyrokinetic model reduces to drift kinetic electrons model for electrons species.

### 2.2. Conservative scheme

In the particle-in-cell method, the electrons’ perturbed density \( \delta n_e \) and the perturbed flow \( \delta u_{i|e} \) are calculated with large numerical noises, so \( \delta n_e \) and \( \delta u_{i|e} \) fail to satisfy the electron continuity equation accurately. As a result, the inductive and electrostatic parallel electric fields do not cancel with each other well, leading to an unphysical component in the parallel electric field \( \nabla \mathbf{E}_p \). To solve this, we use the conservative scheme [11, 12], where the particle gathering process is only taken for the electrons pressure terms \( \delta P_{i|e} \) and \( \delta P_{e|e} \). \( \delta n_e \) and \( \delta u_{i|e} \) are solved from electron continuity equation and the Ampere’s law for consistency.

Using the equation (7), \( \delta v_e \) is time advanced and used to calculate the electron pressure terms \( \delta P_{i|e} \) and \( \delta P_{e|e} \). For the electron density, we integrate the equation (7) to get the electron continuity equation:

\[
\frac{\partial \delta n_e}{\partial t} = - \mathbf{B}_0 \cdot \nabla \left( \frac{n_e \delta u_{i|e}}{B_0} \right) - \mathbf{B}_0 \cdot \nabla v_E \cdot \nabla \left( \frac{n_e + \delta n_e}{B_0} \right) + n_{0e} (\delta v_{i|e} + v_E) \cdot \nabla \left( \frac{\delta P_{i|e}}{B_0} \right) - \delta \mathbf{B}
\]

(7)

\[
\begin{align*}
\nabla \left( \frac{n_e \delta u_{i|e}}{B_0} \right) - c \nabla \left( \frac{\delta P_{i|e}}{B_0} \right) - c \nabla \mathbf{B}_0
\end{align*}
\]

(10)

Here \( \delta v_{i|e} \) is the perturbed electron diamagnetic drift velocity defined as \( \delta v_{i|e} = \frac{1}{n_{0e} m_{i|e}} \mathbf{B}_0 \times \nabla (\delta P_{i|e} + \delta P_{e|e}) \). The electron parallel flow \( n_{0e} \delta u_{i|e} \) is calculated by inverting the Ampere’s law equation (9):

\[
Z_e n_{0e} \delta u_{i|e} = -\frac{4\pi}{c} \nabla^2 \delta A_i - Z_i n_{0i} \delta n_i.
\]

(11)

The electrostatic potential \( \delta \phi \) is solved by the gyrokinetic equation (8):

\[
Z_e^2 \frac{n_0}{T_0} (\delta \phi - \delta \phi) = Z_i \delta n_i + Z_e \delta n_e.
\]

(12)

Taking the time derivative for the Ampere’s law (9), one can get the Ohm’s law for \( \delta A_i \):

\[
\nabla^2 \frac{\partial \delta A_i}{\partial t} = -\frac{4\pi}{c} Z_e n_{0e} \frac{\partial \delta u_{i|e}}{\partial t}
\]

(13)

Note that \( \frac{\partial \delta u_{i|e}}{\partial t} \) is much smaller than \( \frac{\partial \delta u_{i|e}}{\partial t} \) due to the small mass ratio \( m_{e|e} / m_i \ll 1 \) and is thus dropped for simplicity. For the RHS of the equation (13), we can integrate the drift kinetic equation (7) with \( v \parallel \) to get the electron momentum equation:

\[
\begin{align*}
\frac{\partial \delta u_{i|e}}{\partial t} & = \left\{ \frac{B_0 + \delta \mathbf{B}}{m_{i|e}} \cdot \left[ \nabla \delta P_{i|e} + (\delta P_{i|e} - \delta P_{i|e}) \frac{\nabla \mathbf{B}_0}{\mathbf{B}_0} \right] + \frac{\delta \mathbf{B}}{m_{i|e}} \cdot \left[ \nabla \delta P_{i|e} + (\delta P_{i|e} - \delta P_{i|e}) \frac{\nabla \mathbf{B}_0}{\mathbf{B}_0} \right] \right. \\
& \quad + \frac{c}{B_0} \mathbf{B}_0 \cdot \left[ \mathbf{B}_0 \cdot \nabla \delta \phi + (n_{0e} + \delta n_e) \delta \mathbf{B}_0 \right] - \left. 3 n_{0e} \frac{\nabla \mathbf{B}_0}{\mathbf{B}_0} \right\} \\
& \quad + \frac{c}{B_0} \mathbf{B}_0 \cdot \left[ \nabla \delta \phi + 3 \nabla (n_{0e} \mathbf{T}_{0e} u_{i|e}) \right] - \left. n_{0e} \frac{\nabla \mathbf{B}_0}{\mathbf{B}_0} \right\}
\end{align*}
\]

(14)

where \( u_{i|e} = (\delta u_{i|e} + u_{i|e}) \). The energy flux terms are much smaller than other terms in the equation (14), so we ignore kinetic effects in these terms to reduce particle noises by...
using the following closures [30]: \( \delta \psi = 3n_0 e T_{0e} \delta \zeta_{\psi} \), \( \delta \psi_0 = n_0 e T_{0e} \delta \zeta_{\psi_0} \). Here we define:

\[
\Delta = - \frac{4\pi Z_a n_0 e \partial \delta u_{\parallel e}}{\omega_{pe}^2} \frac{\partial \delta A_i}{\partial t} + \frac{\partial \delta A_i}{\partial \zeta},
\]

(15)

where \( \omega_{pe} = \sqrt{\frac{4\pi Z_a e^2 n_0}{m_e}} \) is the electron plasma frequency, the Ohm’s law (13) can be written as:

\[
\left( \nabla^2 - \frac{\omega_{pe}^2}{c^2} \right) \frac{\partial \delta A_i}{\partial t} = \frac{\omega_{pe}^2}{c^2} \Delta.
\]

(16)

Combining the equations (7), (10)–(16) the system is closed and can be used for nonlinear electromagnetic simulations.

The kinetic electron effects in the conservative scheme come from the pressure terms: \( \delta P_{\parallel,e}, \delta P_{\perp,e} \), which can be reduced to the fluid electron model if desirable. For example, one can take isothermal approximation for the electrons pressure

\[
\delta P_{\parallel,e} = \delta P_{\perp,e} = \delta n_e e T_{0e} + n_0 e \left( \frac{\partial T_{0e}}{\partial \zeta_{\psi}} \delta \psi + \frac{\partial T_{0e}}{\partial \zeta_{\theta}} \delta \theta \right).
\]

(17)

where \( \delta \zeta_{\theta} \), \( \delta \psi \) are used for the Clebsch representation of the magnetic field: \( B_0 = \nabla \zeta_{\psi} \times \nabla \zeta_{\theta} \) and \( B_0 + \delta B = \nabla (\psi_0 + \delta \psi) \times \nabla (\theta_0 + \delta \theta) \).

This conservative model is different from the previous conservative model [11]. The previous model splits the electron response into an adiabatic part plus a non-adiabatic correction and solves the zonal and non-zonal components of perturbations separately, the adiabatic part is treated with fluid equations and the non-adiabatic is solved dynamically. On the other hand, our model solves the zonal and non-zonal components together, which is required for simulations with magnetic islands. Finally, the isothermal approximation for the electrons pressure in the previous conservative model [11] is

\[
\delta P_{\parallel,e} = \delta P_{\perp,e} = e n_0 T_{0e} \delta \psi + \frac{\partial T_{0e}}{\partial \zeta_{\psi}} \delta \psi + \frac{\partial T_{0e}}{\partial \zeta_{\theta}} \delta \theta, \quad \delta \psi^A, \delta \theta^A \text{ are the adiabatic components of } \delta \psi, \delta \theta.
\]

3. The interactions between an \( n = 1, m = 2 \) magnetic island and ITG

Previous studies based on slab-geometry have suggested that magnetic islands can modify the ITG mode structures [21–25]. In particular, the linear eigenfunction of the ITG mode is shifted towards the X-point of the magnetic island [21, 25], some nonlinear works found that a magnetic island induced ITG mode (MITG) [22–24] can be excited in the nonlinear stage, where the ITG structure is spread along the magnetic island separatrix. In this section, we use our model to verify these observations in a tokamak geometry. The conservative model using the kinetic electrons still has numerical issues when the plasma beta is small (i.e. ITG case), and this is being investigated and will be reported in a future publication. For simplicity, we use fluid electrons in this part, where the isothermal approximation (17) is used.

Figure 1. Poloidal contour plot of the helical flux \( \psi_e \) of the (2,1) magnetic island at the \( \zeta = 0 \) poloidal plane.

The simulation parameters are as follows: inverse aspect ratio \( a/R_0 = 0.42 \), where \( a \) is minor radius and \( R_0 \) is major radius, ion gyro-radius \( \rho_i = 2.86 \times 10^{-3} R_0 \). At the magnetic axis, \( T_e = T_i = 2.22 \text{ keV}, n_e = n_i = 1.13 \times 10^{13} \text{ cm}^{-3} \).

The electron temperature profile is uniform but the ITG is kept for the ITG drive. At the magnetic surface with \( r = 0.5a, q = 2, \delta = 0.54 \). Defining the characteristic lengths of the density and temperature gradients as:

\[
L_n = \frac{n_0}{\partial n_e/\partial r}, \quad L_T = \frac{T_0}{\partial T_{0e}/\partial r}, \quad r = 0.5a, R_0/L_m = 1.9, R_0/L_T = 6.0 \text{ and } R_0/L_{ne} = 1.9, R_0/L_{Te} = 0.0.
\]

We keep in simulation only 4 toroidal modes with \( n = 1, m = 12, 13, 14 \), where the \( n = 1 \) component relates to the magnetic island harmonic and the \( n = 12, 13, 14 \) parts are the most unstable ITG harmonics.

We introduce a prescribed \( n = 1, m = 2 \) magnetic island in our simulation by adding an external parallel vector potential \( \delta A_{\parallel IS} = -A_{\parallel 0} R_0 \sin(2(\zeta - \Delta) \times h(r - r_0)) \) to the equilibrium. The perturbed magnetic field is:

\[
\delta B_{\parallel IS} = \nabla \times \delta A_{\parallel IS} b_0.
\]

(18)

Here, \( A_{\parallel 0} \) is a parameter that controls the island’s size, \( h(r - r_0) \) is a radial envelope function used to suppress \( \delta A_{\parallel IS} \) at the simulation boundary, and \( r_0 = 0.5a \). \( h(r - r_0) \) has the following form:

\[
h(r - r_0) = \left( 1 + \tanh \left( \frac{2r - r_0}{r_0} \right) \right) \left( 1 + \tanh \left( \frac{r_0 - r}{r_0} \right) \right).
\]

(19)

The width of the magnetic island is estimated by:

\[
W = \sqrt{\frac{8\pi A_{\parallel 0} R_0}{B_0^2 \delta}}.
\]

(20)

For our case, the calculated magnetic island width is \( W = 0.13a = 22 \rho_i \). The island topology can be viewed from
a magnetic helical flux function [25] defined as:

\[ \psi_{he} = \psi - \psi_i - \frac{\delta A_i}{B_0} g, \]  

(21)

where \( \psi, \psi_i \) are the magnetic poloidal flux and the toroidal flux functions, and \( g \) is a coefficient from the covariant representation of the equilibrium magnetic field: \( B_0 = I \nabla \theta + g \nabla \zeta \). The contour plot of \( \psi_{he} \) on the poloidal plane at \( \zeta = 0 \) is shown in figure 1.

The (2,1) magnetic island flattens the plasma profile by following equation, where the ITG drive is removed:

\[ \frac{d}{dt} \omega = -(1-w) \times \left( \frac{\delta B_{n0}}{B_0} \cdot \kappa \right). \]  

(22)

We use the equation (22) to evolve the system for 20R_0/C_s until it achieves a flattened state, \( C_s = \sqrt{\frac{P_{\text{be}}}{n_0}} \) is the ion sound speed. In figure 2 (panel (a)), when \( t = 20R_0/C_s \), we plot the radial profile of the ion density at the strong field side (\( \theta = \pi, \zeta = 0 \)). The magnetic island boundary is marked by the green dashed lines, and it is clear that the total ion density profile (red line) is flattened inside the magnetic island. In figure 2 (panel (b)), we also plot the radial gradient of the ion density at the magnetic island O-point (\( \theta = \pi, \zeta = 0, r = 0.5a \)), which is gradually decreased to zero in the flattening process.

After we get the flattened profiles at \( t = 20R_0/C_s \), we set it as the initial state of the system and use the conservative scheme to evolve the system self-consistently. In this stage, turbulence drive is turned on in the weight equation (7) and the magnetic island is evolved using the Ohm’s law (16). Note that in figure 2 the perturbations of particle densities are large for the flattened state (\( \delta n/n_0 \sim 10^{-1} \)), to ensure the quasineutral condition, we enforce \( \delta n_i = \delta n_e \) at \( t = 20R_0/C_s \).

The magnetic islands can rotate with electrons due to the frozen-in-line effect. The characteristic rotation frequency is the electron diamagnetic frequency [19, 20]. This phenomenon is successfully verified both in our linear and nonlinear simulations. In figure 3, we plot the poloidal contour plot of \( \delta A_i \) at \( t = 20R_0/C_s \) and \( t = 60R_0/C_s \). It is clear that \( \delta A_i \) is dominated by the magnetic island induced component which has a \( m = 2 \) structure. The magnetic island is found to rotate at the electron diamagnetic direction, from figures 3(a) to (b), the island structure rotates about \( \pi/3 \) in 40R_0/C_s, with a measured poloidal drift velocity of \( v = \frac{\pi R_0}{120C_s}/R_0 \). The theoretical rotation frequency can be estimated from the Ohm law (16) by keeping the dominant linear terms:

\[ \frac{\partial \delta A_i}{\partial t} = -c b_0 \cdot \nabla \left( \frac{\delta P_{\text{in}}}{Z_e n_{0e}} + \delta \phi \right) + c Z_e n_{0e} b_0 \cdot \nabla \delta A_i. \]  

(23)

Note that \( v_{se} = -c Z_e n_{0e} b_0 \) is the electron diamagnetic drift velocity. If we take the limit of long-wavelength to the parallel direction, we have:

\[ \frac{\partial \delta A_i}{\partial t} \approx -v_{se} \cdot \nabla \delta A_i. \]  

(24)

Taking \( \nabla = ik_e e_\zeta + ik_\theta e_\theta \) and \( \frac{\partial}{\partial t} = -i \omega \) for the perturbed quantities. Where \( e_\zeta \) and \( e_\theta \) are the unit vectors of the toroidal direction and the poloidal direction. We have

\[ \omega \approx k_{\theta} |v_{se}|. \]  

(25)

The estimated theoretical rotation frequency is \( \omega = 0.057C_s/R_0 \), which is close to the measured rotation frequency is \( \omega = k_{\theta} v = 0.052C_s/R_0 \).

We further find that the ITG mode structures are significantly modified by the magnetic island. Due to the initially large perturbations, we cannot observe a linear to nonlinear transition in a single simulation, so we separately carry out linear and nonlinear simulations. The nonlinear terms in the weight equation (7) and the Ohm’s law (16) are closed in the linear simulation, while the island impact in the equations of motion (2), (3) is kept in both two simulations. In the linear simulation, at \( t = 60R_0/C_s \), we plot the poloidal structure of \( \delta \phi^{n+1} = \delta \phi - \delta \phi^{n-1} \), which represents the high n components of the ITG electrostatic potentials. In figure 4, in the linear case (panel (a)), the ITG poloidal mode structure is decreased at the island O-point and peaked at the X-point.
Here the ITG structure is still located around the $q = 2$ resonant surface ($r = 0.5a$). Our result agrees with a linear theoretical study, which argues that the magnetic island flattening effects can decrease the ITG drive at the island O-point [21]. Consequently, the ITG mode structure is shifted to the X-point.

In the nonlinear case (panel (b)), we also find that the ITG fluctuation is decreased at the island O-point, but more interestingly, the nonlinear ITG structure is spread along the magnetic island separatrix. Previous fluid simulations [22–24] based on a slab-geometry configuration also found that MITG can be nonlinearily excited and mainly locates at the island separatrix. Note that in figure 2(a), the flattened ion density profile has a strong gradient at the island separatrix, which strengthens the ITG drive at the separatrix and can be an important factor leading to the ITG structure spreading along the separatrix. Previous slab simulations [22–24] also suggested that the magnetic island induces a broadened distribution of the rational surface (i.e. $k_\parallel = 0$ at the separatrix), which causes the ITG mode excitation at the island boundary. In this paper, we used our model to verify the magnetic island effects on ITG turbulence both linearly and nonlinearly.

Figure 3. Poloidal contour plot of perturbed magnetic vector potential $\delta A_1$ at (a) $t = 20R_0/C_s$ and (b) $t = 60R_0/C_s$.

Figure 4. Poloidal contour plots for the $n > 1$ components of the electrostatic potential $\delta \phi^{n>1}$. (a) The linear simulation, (b) the nonlinear simulation.
mainly through the change of local instability drive (i.e. change of ion pressure profile) and the existence of island magnetic surfaces. However, it should be emphasized that magnetic islands and microturbulence can interact with each other in other ways. For example, recent experiments and global gyrokinetic simulations both demonstrate that large scale mode and vortex shear flow can be excited around magnetic islands [4, 7, 31], which play an important role in the regulation of turbulence and radial transport near the island. On the other hand, turbulence spreading outside of magnetic island into the O-point of the island can influence NTM dynamics [33]. In the future, we will use kinetic electrons in our model to further study those important physics to comprehensively understand nonlinear interactions between magnetic islands and microturbulence.

4. Conclusion

In conclusion, we extend the conservative scheme of kinetic electrons for gyrokinetic simulations [11, 12] in the presence of magnetic islands in the GTC, where zonal and non-zonal components of all perturbed quantities are solved together. Our conservative model uses electron continuity equation and the Ampere’s law to evolve electron density and parallel flow, thus enforcing the consistency in the calculation of parallel electric field. By default, our model uses gyrokinetic ions and drift-kinetic electrons, and the model can be reduced to a fluid electron model if desirable. We verify this new conservative model for drift-Alvénic waves with kinetic electrons and use it to study the interactions between magnetic islands and microturbulence. In particular, simulation results of the KAW in a slab-geometry agree well with the theoretical dispersion relation. We also demonstrate superior numerical properties of our conservative scheme when simulation grid size is larger than electron skin depth. Secondly, we benchmark the conservative model with the electromagnetic hybrid model [14] for linear KBM. In the fluid electrons simulations, we find that the two models get very good agreement for the KBM growth rates and real frequencies. For the kinetic electrons simulations, the two models also get reasonable agreement. Finally, we use the conservative model with fluid electrons to study the interactions between a magnetic island and the ITG. In our simulations, we find that the magnetic island rotates at the electron diamagnetic drift velocity, which is consistent with the theory of the drift-tearing mode [19, 20]. We also observe that the magnetic island can significantly modify the ITG mode structures. In the linear simulation, the ITG structure shifts from the island O-point toward the island X-point due to the pressure flattening inside the island. In the nonlinear simulation, the ITG structure peaks along the magnetic island separatrix because of the increased pressure gradient there. Our results are in qualitative agreement with previous theoretical and numerical studies of the magnetic island effects on the ITG mode [21–25].

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Appendix A. Verification of KAW

We first use the conservative scheme to simulate the KAW in a shear-less cylindrical geometry in uniform plasmas. For the simulations in this sub-section ions only provide polarization density for simplicity. In the linear case, the dynamic equation (7) for electron can be simplified as:

\[
\left( \frac{\partial}{\partial t} + v_b \cdot \nabla \right) \delta f_e = - \frac{e}{m_e} \nabla \delta \phi + \frac{1}{c} \frac{\partial \delta A_{\parallel}}{\partial t} \frac{\partial \delta n_e}{\partial v_f}.
\]

Equations (8)–(10), (16) become:

\[
\frac{c^2}{4 \pi e V_A^2} \nabla^2 \phi = \delta n_e = 0,
\]

\[
n_0 e \delta n_{i, e} = \frac{c}{4 \pi e} \nabla^2 \delta A_{\parallel},
\]

\[
\frac{\partial \delta n_e}{\partial t} = - b_0 \cdot \nabla (n_0 e \delta n_{i, e}),
\]

\[
\left( \nabla^2 - \frac{\omega_{pe}^2}{c^2} \right) \frac{\partial \delta A_{\parallel}}{\partial t} = \frac{\omega_{pe}^2}{c} b_0 \cdot \nabla (\delta \phi - \delta P_{\perp e} / n_0 e).
\]

Here, \( V_A = \frac{B_0}{\sqrt{4 \pi n_0 m_e}} \) denotes the Alfvén speed. Applying \( \frac{\partial}{\partial t} = - i \omega_t, \nabla \cdot i k_z, b_0 \cdot \nabla = i k_z \) to the above equations, we can get the linear dispersion relation of KAW as:

\[
\left( \frac{\omega^2}{k^2 V_A^2} - 1 \right) \left[ 1 + \xi_e Z(\xi_e) \right] = k_z^2 \rho_s^2.
\]

Here, \( \xi_e = \frac{\omega}{\sqrt{2} \omega_{pe}}, \rho_s = \sqrt{\frac{m_e}{m_i}} / \Omega_{ci} \) and \( Z(\xi_e) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-x^2} dx \) is the plasma dispersion function.

The parameters used to simulate the KAW with kinetic electrons are set as follows: the electron temperature \( T_{ke} = 5.0 \text{ keV} \), the magnetic field \( B_0 = 1.5 \text{ T} \), and the ratio between the perpendicular wave vector and the parallel wave vector is \( k_z / k_\perp = 0.01 \). Figure A1 shows that the simulation...
results from the conservative scheme (red cross) agree well with the theoretical predictions (solid lines) both for the real frequency $\omega$ (panel (a)) and the damping rate $\gamma$ (panel (b)) when we change the equilibrium $\beta_e = \frac{8n_e \rho_i}{d_e}$. The advantage of the conservative scheme [11, 12] is that it can avoid the requirement of grid size being larger than electron skin depth by using the continuity equation to guarantee the consistency between the electron density and flow. To illustrate this, we compare the original method using kinetic markers to calculate electron density and the conservative scheme in this paper. For the two models, we use an antenna with the theoretical frequency to excite the KAW and vary the grid size from $\Delta x = D_{0.38}$ to 7.6, here $D_{0.38}$ is electron skin depth size and $\Delta x$ is perpendicular grid size. In figure A2, panel (a) shows that when the grid size is smaller than the electron skin depth ($\Delta x = 0.38$), the two models both recover KAW oscillations. However, when we increase the grid size ($\Delta x = 3.8$ (panel (b)) and $\Delta x = 7.6$ (panel (c)), the conservative scheme can recover KAW correctly, but the original method without the continuity equation fails due to the inconsistency between the electron density and flow.

Appendix B. Verification of KBM

We next verify the conservative model for linear KBM in tokamak geometry, the simulation parameters used in this part are set as follows: inverse aspect ratio $a/R_0 = 0.42$, where $a$ is the minor radius and $R_0$ is major radius, and ion gyro-radius $\rho_i = 2.86 \times 10^{-3} R_0$. The electron temperature profile is uniform but the ITG is kept for the KBM drive. At the magnetic axis, $T_e = T_i = 2.22$ keV, $n_e = n_i = 9.0 \times 10^{13}$ cm$^{-3}$. On the magnetic flux surface with $r = 0.5a$, $q = 2$, and the magnetic shear $s = 0.54$. Defining the characteristic lengths of the density and temperature gradients as: $d_n = \frac{n_i}{d_e}$, $d_T = \frac{T_i}{d_e}$ at the surface $r = 0.5a$, for the ion $R_0/L_n = 1.9$, $R_0/L_T = 6.0$ and for the electron $R_0/L_n = 1.9$, $R_0/L_T = 0.0$.

The electromagnetic hybrid model [14] in GTC is used for benchmark. In linear case, our model keeps linear terms in the equation (14) and uses the equation (16) to calculate the inductive electric field, while the hybrid model [14] uses $\frac{\partial E_i}{\partial t} = -b_d \cdot \nabla (b_0 \hat{\phi}_{\text{eff}} - \hat{\phi})$, where $b_0 \hat{\phi}_{\text{eff}}$ is an effective potential defined with $\delta E_i = -b_d \cdot \nabla \hat{\phi}_{\text{eff}}$. First we show that the two models would get very close results when the fluid
electrons are used. In the fluid limit, the isothermal approximation (17) is used for the conservative model. We scan the normalized perpendicular wave number $k_{\perp\rho_i}$ from 0.1 to 0.8 to calculate the KBM real frequency and linear growth rate. In figure B1, we observe that the fluid electron conservative model (red dot) gets nearly the same KBM real frequency (panel (a)) and linear growth rate (panel (b)) with the fluid hybrid model (blue dot).

When the kinetic electrons are included (red and blue stars), the KBM real frequencies (figure B1 panel (a)) are very close to the fluid results, this is because that the KBM real frequency is mainly decided by ions dynamics. In figure B1 (panel (b)), the conservative model (red star) and the hybrid model (blue star) get similar results, the linear KBM growth rates are all decreased compared to the fluid case, which is consistent with the previous study that the trapped electrons can reduce the growth rate of the KBM [34]. The $\delta\phi$ and $\delta A_{\parallel}$ poloidal mode structures of $k_{\perp\rho_i} = 0.42$ mode are shown in figure B2 for the kinetic conservative model. The agreements of the two models on the linear KBM physics verify that our conservative scheme can be reliably used to study the drift-wave instability physics.

Figure B1. Simulation results from the conservative scheme (red) and the hybrid model (blue) for KBM real frequency $\omega$ (panel (a)) and linear growth rate $\gamma$ (panel (b)) when $k_{\perp\rho_i}$ is changed.

Figure B2. Poloidal contour plots of the KBM poloidal mode structures in the conservative scheme simulation with $k_{\perp\rho_i} = 0.42$, (a) perturbed electrostatic potential $\delta\phi$ and (b) perturbed electromagnetic potential $\delta A_{\parallel}$.
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