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Unconventional ballooning structures for toroidal drift waves

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With strong gradients in the pedestal of high confinement mode (H-mode) fusion plasmas, gyrokinetic simulations are carried out for the trapped electron and ion temperature gradient modes. A broad class of unconventional mode structures is found to localize at arbitrary poloidal positions or with multiple peaks. It is found that these unconventional ballooning structures are associated with different eigen states for the most unstable mode. At weak gradient (low confinement mode or L-mode), the most unstable mode is usually in the ground eigen state, which corresponds to a conventional ballooning mode structure peaking in the outboard mid-plane of tokamaks. However, at strong gradient (H-mode), the most unstable mode is usually not the ground eigen state and the ballooning mode structure becomes unconventional. This result implies that the pedestal of H-mode could have better confinement than L-mode. \textsuperscript{©} 2015 AIP Publishing LLC.

Although numerous theoretical models have been suggested,\textsuperscript{1} a yet unexplained phenomenon in tokamak fusion plasmas is the transition of low (L) to high (H) confinement states, where H-mode\textsuperscript{2} has significant better confinement property than that of the L-mode. Understanding of the H-mode physics is not only important to make controlled fusion more feasible but also that the existence of and transitions among multi-equilibrium states are important fields of nonlinear physics in laboratory and the Universe. Drift wave turbulence is one of the major causes that leads to the anomalous transport widely observed in fusion and space plasmas.\textsuperscript{3,4} In order to control the turbulent transport, it is crucial to understand the underlying transport mechanism, which may vary for different types of instability that drive the turbulence. The correlation time and length are found to be closely related to the mode structure of the turbulence.\textsuperscript{5} Therefore, the mode structure of the turbulence has a significant effect on the transport level.\textsuperscript{6}

In this letter, we show that the linear properties of two major types of electrostatic micro-instabilities, namely, the trapped electron mode (TEM) and ion temperature gradient (ITG) mode, are completely different in the H-mode (strong gradient) and L-mode (weak gradient) stages. With the conventional weak gradient, the mode structures for drift wave instabilities such as the ITG and TEM are of ballooning type, peaking at the outboard mid-plane of the tokamak (cf. Refs. 7 and 8). This type of solution is the solution localized at the outside mid-plane, i.e., $\theta_p = 0$ in our notation, where $\theta_p$ is defined as the local peak poloidal angle for the mode structure. For this reason, many local eigenvalue codes such as HD7\textsuperscript{12} assume implicitly $\theta_b = 0$. The unconventional eigen modes with $\theta_p \neq 0$ have been recently discovered in the strong gradient parameter regime. Typically, $|\theta_p| \simeq < \pi/2$ have been shown to exist.\textsuperscript{7,8,13,14} In this work, we find the most general unconventional eigen mode structures from first principle gyrokinetic simulations. The underlying physics is also explained and it has important implications for turbulent transport.

We first obtain linear electrostatic results from global gyrokinetic particle simulation using the GTC code\textsuperscript{15,16} with single toroidal mode number $n$. The simulation parameters and profiles are similar to that of the recent H-mode experiments of the HL-2A tokamak\textsuperscript{17} toroidal magnetic field $B_0 = 1.35$ T, minor radius $a = 40$ cm, major radius $R_0 = 165$ cm, safety factor $q = 2.5–3.0$, magnetic shear $s = 0.3–1.0$, and $R_0/qLT = 80–160$ with $T_e(r) = T_i(r)$ and $n_e(r) = n_i(r)$. $L_q = -(1/n)(dh/dr)$ and $L_{qT} = -1/(T)(dT/dr)$ are density and temperature gradient scale length. Typical electron temperature (also density) profiles used in this letter are shown in Fig. 1. We start with $\eta = L_q/L_{qT} = 1.0$ for simplicity. Collisions are included in some cases but shown little influence to the general results. Under these parameters, no instability or only weakly unstable mode can be found when the electrons are adiabatical. Thus, the major instability for these simulation parameters is the trapped electron mode.

These TEM simulations show that both conventional and unconventional ballooning mode structures can exist for various gradients and toroidal mode numbers ($n = 5–30$), as shown by Fig. 2. In these sub-figures, $q$ profiles are similar. For Figs. 2(b)–2(i), the global density (also temperature) profiles and toroidal mode numbers are not the same but all are under strong gradient. The novel features include: (a) the mode can have anti-ballooning structure (i.e., $|\theta_p| > \pi/2$, e.g., Fig. 2(g)); (b) the mode can have multiple peaks (e.g., Fig. 2(b)). Considering that the trapped particles are mainly

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located at the low magnetic field side, i.e., the outboard side, the anti-ballooning structures of TEM are not expected. The 3D mode structure of the electrostatic potential in (X,Z) plane for TEM observed in GTC simulation, where (a) uses HL-2A tokamak edge weak gradient L-mode plasma parameter (R_0/\Delta_L < 40) and (b)–(i) use edge strong gradient H-mode parameters (R_0/\Delta_L > 80). Collisions are only included in (e) and (g).

FIG. 2. Conventional (a) and unconventional (b)–(i) 2D ballooning structures of electrostatic potential in (X,Z) plane for TEM observed in GTC simulation, where (a) uses HL-2A tokamak edge weak gradient L-mode plasma parameter (R_0/\Delta_L < 40) and (b)–(i) use edge strong gradient H-mode parameters (R_0/\Delta_L > 80). Collisions are only included in (e) and (g).

FIG. 3. The real part of Fourier $\delta \phi_m(r)$ for conventional and unconventional mode structures. The corresponding poloidal cross section mode structures of (a)–(d) (n = 20, 10, 5, 10, respectively) are taken from Figs. 2(a), 2(b), 2(g), and 2(i), respectively. The dashed lines are corresponding rational surface positions $r_s$, where $nq(r_s) = m$.

by Figs. 2(b)–2(d). Under stronger gradients, the radial peaking position of $\delta \phi_m(r)$ is also not at the corresponding rational surface position $r_s$ any more, where $nq(r_s) = m$. Next, we consider ITG mode by reducing the density gradient to $R_0/L_n < 40$ and keeping the other parameters the same as those for the TEM case, e.g., $R_0/L_T > 80$ and thus $\eta_1 = L_n/L_T > 2.0$. To completely exclude the contribution of the kinetic electrons, we use adiabatic electrons in the simulations. It is found that the preceding unconventional mode structures still exist and exhibit even more structural variations. For example, the anti-ballooning structure is found for this ITG simulation, as is shown in Figs. 4(a) and 4(b). Actually, the mode structure with global profiles and multi modes coexisting in the initial value simulation can be even more complicated. For example, two modes with similar growth rates can be excited in different radial locations, as shown in Figs. 4(c) and 4(d). Multi modes coexist with close peaking positions in the initial value simulation can also lead to $\theta_p = \theta_p(t)$, i.e., rotate poloidally with time. Thus, the unconventional mode structures are not limited to TEM and can be common for drift waves.

These unconventional linear behaviors can be understood from the following eigenmode analysis. We start with the ITG eigen mode equation\(^8\),\(^{10}\)

\[ \left[ \rho_i^2 \frac{\partial^2}{\partial x^2} - \frac{\sigma^2}{\omega^2} \left( \frac{\partial}{\partial \theta} + ik_0 \alpha x \right)^2 - \frac{2 \epsilon_n}{\omega} \left( \cos \theta + i \sin \theta \frac{\partial}{\partial \theta} \right) \right] \delta \phi(x, \theta) = 0, \tag{1} \]

where $\sigma = e/\epsilon_\rho (q k_0 \rho_i)$, $\epsilon_n = L_n/R_0$, $n_\parallel = 1 + \eta_1$, $x = r - r_s$, $r_s$ is the rational surface, $\omega = \omega_0 + i \gamma$ is the complex mode frequency normalized by the electron diamagnetic frequency, and the poloidal wave number $k_0 = nq r_s$. Eq. (1) can be derived from the gyrokine theory with adiabatic electron
Eq. (1) can be rewritten as the 2D eigenmode equation

\[
\frac{\sigma^2}{\omega^2} \frac{d^2}{d\theta^2} + k_0^2 \phi_1^2 \left[ 1 + s^2 (\vartheta - \vartheta_1)^2 \right] \\
+ \frac{2\varepsilon_n}{\omega} \cos \theta + s(\vartheta - \vartheta_1) \sin \theta + \frac{\omega - 1}{\omega + \eta_1} \delta \phi(\vartheta, \vartheta_1) = 0,
\]

where \(\vartheta_1\) is the ballooning-angle parameter, which represents an as yet undefined radial wavenumber.\(^{10}\) The relation between the ballooning space electrostatic potential \(\delta \phi(\vartheta, \vartheta_1)\) and real space \(\delta \phi(x, \theta)\) can be found in Refs. 8 and 10. Using the Fourier basis \(\delta \phi(x, \theta) = \sum_m \delta \phi_m e^{im\theta}\), Eq. (1) can be rewritten as the 2D eigenmode equation

\[
k_0^2 s^2 \frac{d^2}{d\theta^2} \frac{\sigma^2}{\omega^2} (z - m)^2 \phi_m - \frac{\omega - 1}{\omega} \left( 1 - s \frac{\partial}{\partial z} \right) \phi_{m-1} \\
+ \left( 1 + s \frac{\partial}{\partial z} \right) \phi_{m+1} - \left( \frac{\omega - 1}{\omega + \eta_1} + k_0^2 \right) \phi_m = 0,
\]

where \(z = k_0 x\). To solve the eigenvalue problem of Eq. (3), only a few number of \(m\) modes need to be kept for the solution to reach convergence.

With suitable approximations (cf. Ref. 22), both Eqs. (2) and (3) can be reduced to the Weber equation \(u'' + (b x^2 + a)x = 0\) (here, the argument \(x\) is \(\vartheta\) and \(z\) for Eqs. (2) and (3), respectively), which has solutions with the eigenvalues \(a(\omega) = i(2l + 1) \sqrt{b(\omega)}\) and eigenfunctions \(u(x) = H_l(i \sqrt{b} x)e^{-ibx^2/2}\), where \(H_l\) is \(l\)-th Hermite polynomial and \(l = 0, 1, 2, \ldots\), which represent a series eigenstates. With the original equations, i.e., Eqs. (2) and (3), which can only be solved numerically, the eigenstates take a more complicated form.
solutions of Eq. (3). Two examples are shown in Fig. 6. The condition for the jump of the most unstable solution can shift from ground state to other non-ground states, which is analogous to the quantum jump behavior between energy levels. Physically, the jump behavior can be understood from the effective potential. The jump happens between energy levels. It is not transparent that the non-ground eigen state solved from Eq. (3) can form the unconventional mode structure in the 2D poloidal plane. Next, we confirm this link by showing that the non-ground 2D eigen states in Refs. 7, 8, and 13 are just weak asymmetric solutions of our series solutions. Almost all the mode structures observed in the preceding gyrokinetic simulation. The solutions in Refs. 7, 8, and 13 are just weak asymmetric solutions of our series solutions. Almost all the mode structures in Figs. 2 and 4 have also been found in both 2D eigen solver and GTC initial simulations. The condition for the jump of the most unstable eigen state to non-ground state is $\epsilon_n < \epsilon_r$, where $\epsilon_r$ is a critical gradient parameter which depends on other parameters. In GTC simulations of the HL-2A parameters, the typical critical density (or temperature) gradient value is $R_0/L_0 = 40$–120.

The results from the gyrokinetic simulation and eigen mode analysis show that the unconventional mode structures exist mainly in the strong gradient regime or the H-mode. In the weak gradient regime or L-mode, conventional mode structures still prevail. This can indicate different transport behavior between H-mode and L-mode. In the conventional ballooning structure, the neighboring Fourier modes $\delta \phi_n \approx \delta \phi_{n+1}$, the effective correlation length may be estimated as the width of radial envelope of the modes, say, $\Delta A$. Whereas, in the unconventional ballooning structures, especially for anti-ballooning structure, $\delta \phi_n \approx -\delta \phi_{n+1}$ can occur, i.e., a 180° phase shift for the neighboring Fourier modes, which can change the effective correlation length to the distance of neighboring mode-rational surfaces $\Delta r$. Considering that $\Delta r \ll \Delta A$, we can expect that the H-mode can have better confinement.

To summarize, a broad class of unconventional ballooning modes is found for electrostatic drift waves (TEM and ITG) by the gyrokinetic simulation, which is shown to be common in the strong gradient regime. These unconventional mode structures are shown to correspond to the non-ground-state solutions of the eigen mode equation. These results may have important implications for the turbulent transport in tokamaks, i.e., the turbulent transport mechanism in the H-mode can be rather different from that in the L-mode, which requires further investigation by self-consistent nonlinear gyrokinetic simulations.

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