Nonlinear fluid closure: Three-mode slab ion temperature gradient problem with diffusion

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The three-mode slab ion temperature gradient problem was considered. Starting from the drift kinetic equation with nonlinear term and diffusion, the hierarchy of fluid equations up to fourth moment was developed. As a closure, the nonlinear fluid closure by N. Mattor and S. Parker [Phys. Rev. Lett. **79**, 3419 (1997)] was applied. Numerical solutions of the system of fluid equations have been obtained and analyzed. The time evolution of electrostatic potential shows that nonlinear fluid closure is able to capture particle trapping, which is important for fusion plasmas. Great attention was paid to studies of the role of diffusion. Diffusion here represents effects of background turbulence and can be described by a Fokker–Planck operator [A. Zagorodny and J. Weiland, Phys. Plasmas **6**, 2359 (1999)]. The three wave system can be considered as a system of test waves in a turbulent background. This system can be used to study situations of varying partial coherence. [DOI: 10.1063/1.1459710]

I. INTRODUCTION

In Ref. 1 Mattor and Parker proposed a nonlinear fluid closure for the description of three-wave interaction of drift waves. The main reason for their work was the different results obtained by different fluid closures in Cyclone work.[2] The key point of the developed approach is to find the formal nonlinear solution of the appropriate kinetic equation (for the given number of interacting modes) and then to establish the relations between the fluid quantities and the higher moments using the obtained formal solution. In Ref. 1 the specific calculations were performed on the basis of the drift-kinetic equation in the collisionless approximation, i.e., disregarding the influence of stochastic turbulent fields on particle trajectories. Obviously, this approximation is no longer valid for the case of well-pronounced turbulence (see, Ref. 3), which produces diffusion in the real and velocity spaces.[3,4] In some cases, such diffusion can be time-nonlocal (non-Markovian).[4,5] So, the questions arise, what is the influence of turbulent diffusion on the resonant interaction of drift waves and what are the main consequences of such influence? The purpose of the present contribution is to extend the fluid closure proposed in Ref. 1 to the description of the three-mode ion temperature gradient (ITG) problem in the presence of turbulent diffusion produced by the stochastic turbulent fields. In order to simplify the problem as much as possible we restrict ourselves to the case of ordinary (Markovian) diffusion in real space.

II. DERIVATION OF FLUID HIERARCHY

We begin with the drift kinetic equation with Fokker–Planck operator on right-hand side, describing the evolution of plasma in a strong magnetic field (see, e.g., Ref. 5),

$$\frac{\partial f}{\partial t}+(\mathbf{v}_{\parallel}+\mathbf{v}_E)\boldsymbol{\nabla} f+\frac{eE_{\parallel}}{m}\frac{\partial f}{\partial v_{\parallel}} =-\beta_i\frac{\partial f}{\partial X_i}+\frac{1}{2}\frac{\partial}{\partial X_i}\left(D_{ij}\frac{\partial f}{\partial X_j}\right), \tag{1}$$

where $f(\mathbf{x},v_{\parallel},t)$ is the one-particle distribution function of ions, $\mathbf{v}_{\parallel}$ is the kinetic velocity parallel to the magnetic field $\mathbf{B}$, $\mathbf{v}_E$ is the $\mathbf{E}\times\mathbf{B}$ velocity, $E_{\parallel}$ is the electric field parallel to $\mathbf{B}$, X is the set of phase variables $\{\mathbf{r},\mathbf{v}\}$, D_{ij} is the diffusion coefficient, and β_i is the viscosity.

The equilibrium has a straight constant magnetic field $\mathbf{B}=B\hat{\mathbf{z}}$ and a Maxwellian distribution of ions with density and temperature gradients in $\hat{\mathbf{x}}$,

$$F_0(x,v_{\parallel})=n_0(2\pi c_s^2/\tau)^{-1/2}\exp[-\tau v_{\parallel}^2/2c_s^2],$$

where $c_s^2=T_e/m_i$, $\tau=T_e/T_0$, T_0 and T_e are the ion and electron temperatures, respectively.

The total one-particle distribution function of ions takes a form

$$f(\mathbf{x},v_{\parallel},t)=F_0(x,v_{\parallel})+\tilde{f}(\mathbf{x},v_{\parallel},t).$$

We also assume quasineutrality, electrostatic field $\mathbf{E}=-\boldsymbol{\nabla}\tilde{\phi}$, and isothermal electrons (Boltzmann response) $\tilde{\phi}(x,t)T_e/e=\tilde{n}(x,t)/n_0$. We restrict ourselves by considering only diffusion and viscosity in real space in the direction perpendicular to the magnetic field and with constant coefficients. It was done in order to simplify the problem as much as possible, however, collisional dissipation, namely, velocity space diffusion, can also be important (see Ref. 6). As in Ref. 1, we neglect nonlinear parallel acceleration.

The kinetic equation (1) Fourier–Laplace transformed in $\mathbf{x}$ and t then reduces to

$$-i(\omega+i\Delta_{\mathbf{k}}-k_{\|}v_{\|})\tilde{f}_{\mathbf{k}}+\frac{1}{B}\sum_{\mathbf{k}'}(k_y''k_x'-k_x''k_y')\tilde{\phi}_{\mathbf{k}''}\tilde{f}_{\mathbf{k}'}$$
$$+i\left[\tau k_{\|}v_{\|}+\omega_{*e}-\omega_{*e}\eta_i\left(\frac{1}{2}-\frac{\tau v_{\|}^2}{2c_s^2}\right)\right]\frac{F_0\tilde{n}_{\mathbf{k}}}{n_0}$$
$$=-\beta_x\frac{\partial F_0}{\partial x}+\frac{1}{2}D_x\frac{\partial^2 F_0}{\partial x^2}, \tag{2}$$

where $\mathbf{k}''=\mathbf{k}-\mathbf{k}'$, $\omega_{*e}=-k_yT_e/(eBL_n)$, $L_n^{-1}=d\ln n_0/dx$, $L_T^{-1}=d\ln T_0/dx$, $\rho_s=T_e/(eBc_s)$, $\eta_i=L_n/L_T$, and $\Delta_{\mathbf{k}}\equiv i(\beta_xk_x+\beta_yk_y)+(D_xk_x^2+D_yk_y^2)/2$. We regard Eq. (2) as exact for the purpose of deriving the hierarchy of fluid equations.

According to Ref. 1 the highest order of velocity in the kinetic equation defines the order of fluid moment on which we should perform nonlinear kinetic closure; in our case the term $\partial^2F_0/\partial x^2$ gives us the fourth order of $v_{\|}$.

We define the dimensionless fluid moments as

$$\tilde{n}_{\mathbf{k}}=\frac{\int dv_{\|}\tilde{f}_{\mathbf{k}}}{n_0},$$
$$\tilde{V}_{\|\mathbf{k}}=\frac{\int dv_{\|}v_{\|}\tilde{f}_{\mathbf{k}}}{n_0c_s},$$
$$\tilde{P}_{\|\mathbf{k}}=\frac{\int dv_{\|}v_{\|}^2\tilde{f}_{\mathbf{k}}}{n_0c_s^2},\quad P_{\|}=nT_{\|}, \tag{3}$$
$$\tilde{Q}_{\|\mathbf{k}}=\frac{\int dv_{\|}v_{\|}^3\tilde{f}_{\mathbf{k}}}{n_0c_s^3},$$
$$\tilde{J}_{\|\mathbf{k}}=\frac{\int dv_{\|}v_{\|}^4\tilde{f}_{\mathbf{k}}}{n_0c_s^4}.$$

The corresponding hierarchy of fluid equations is

$$\frac{\partial\tilde{n}_{\mathbf{k}}}{\partial t}+ik_{\|}\tilde{c}\tilde{V}_{\|\mathbf{k}}+i\tilde{n}_{\mathbf{k}}=-\Delta_{\mathbf{k}}\tilde{n}_{\mathbf{k}}-\beta_x\frac{\rho_s}{L_n}+\frac{D_x}{2}\left[\frac{\rho_s}{L_n}\right]^2, \tag{4}$$

$$\frac{\partial\tilde{V}_{\|\mathbf{k}}}{\partial t}+ik_{\|}\frac{\tilde{c}}{\tau}[\tau\tilde{T}_{\|\mathbf{k}}+(1+\tau)\tilde{n}_{\mathbf{k}}]+\tilde{c}\sum_{\mathbf{k}'}(k_y''k_x'-k_x''k_y')\tilde{n}_{\mathbf{k}''}\tilde{V}_{\|\mathbf{k}'}$$
$$=-\Delta_{\mathbf{k}}\tilde{V}_{\|\mathbf{k}}, \tag{5}$$

$$\frac{\partial\tilde{T}_{\|\mathbf{k}}}{\partial t}+ik_{\|}\frac{\tilde{c}}{\tau}[\tau\tilde{Q}_{\|\mathbf{k}}-\tilde{V}_{\|\mathbf{k}}]$$
$$+\tilde{c}\sum_{\mathbf{k}'}(k_y''k_x'-k_x''k_y')\tilde{n}_{\mathbf{k}''}\tilde{T}_{\|\mathbf{k}'}+i\tilde{n}_{\mathbf{k}}\frac{\eta_i}{\tau}$$
$$=-\Delta_{\mathbf{k}}\tilde{T}_{\|\mathbf{k}}-\beta_x\frac{\rho_s}{\tau L_T}+\frac{D_x}{2\tau}\left[\frac{\rho_s^2}{L_T^2}+\frac{\rho_s^2}{L_nL_T}\right], \tag{6}$$

$$\frac{\partial\tilde{Q}_{\|\mathbf{k}}}{\partial t}+ik_{\|}\tilde{c}[\tilde{J}_{\|\mathbf{k}}+3\tilde{n}_{\mathbf{k}}]+\tilde{c}\sum_{\mathbf{k}'}(k_y''k_x'-k_x''k_y')\tilde{n}_{\mathbf{k}''}\tilde{Q}_{\|\mathbf{k}'}$$
$$=-\Delta_{\mathbf{k}}\tilde{Q}_{\|\mathbf{k}}, \tag{7}$$

$$\frac{\partial\tilde{J}_{\|\mathbf{k}}}{\partial t}+ik_{\|}\tilde{c}\frac{\int dv_{\|}v_{\|}^5\tilde{f}_{\mathbf{k}}}{n_0c_s^5}$$
$$+\tilde{c}\sum_{\mathbf{k}'}(k_y''k_x'-k_x''k_y')\tilde{n}_{\mathbf{k}''}\tilde{J}_{\|\mathbf{k}'}-i\frac{3\tilde{c}}{\tau^2}\left[\frac{\rho_s}{L_n}+\frac{2\rho_s}{L_T}\right]\tilde{n}_{\mathbf{k}}$$
$$=-\Delta_{\mathbf{k}}\tilde{J}_{\|\mathbf{k}}-\frac{3\beta_x}{\tau^2}\left[\frac{\rho_s}{L_n}+\frac{2\rho_s}{L_T}\right]+\frac{3D_x}{2\tau^2}\left[\frac{\rho_s}{L_n}+\frac{2\rho_s}{L_T}\right]^2. \tag{8}$$

Here we introduce the dimensionless parameter $\tilde{c}=c_s/\rho_s\omega_{*e}$. The time and $\mathbf{k}$ vector are normalized by $|\omega_{*e}|^{-1}$ and ρ_s^{-1}, respectively. Closure is now needed, to express $\int dv_{\|}v_{\|}^5\tilde{f}_{\mathbf{k}}$ in terms of lower moments.

III. CLOSURE PROCEDURE

We construct the closure term in the same way as done by Mattor and Parker in Ref. 1. First, we rewrite the kinetic equation (2) in matrix form

$$(\tilde{\Omega}-v_{\|}\mathcal{K}_{\|}-\Gamma)\vec{f}=\Omega_*(v_{\|})F_0. \tag{9}$$

Here $\vec{f}$ is a column vector of $\tilde{f}_{\mathbf{k}}$ for different $\mathbf{k}$, $\tilde{\Omega}$ and $\mathcal{K}_{\|}$ are diagonal matrices of $\omega-i\Delta_{\mathbf{k}}$ and $k_{\|}$, Γ is a matrix of nonlinear coupling coefficients $\Gamma_{\mathbf{kk}'}\equiv i\rho_sc_s(k_y''k_x'-k_x''k_y')\tilde{n}_{\mathbf{k}''}$, and $\Omega_*(v_{\|})$ is the diagonal matrix of

$$\Omega_{*\mathbf{kk}}(v_{\|})=\left[\tau k_{\|}v_{\|}+\omega_{*e}-\omega_{*e}\eta_i\left(\frac{1}{2}-\frac{\tau v_{\|}^2}{2c_s^2}\right)\right]\tilde{n}_{\mathbf{k}}$$
$$+i\frac{\beta_x}{F_0}\frac{\partial F_0}{\partial x}-i\frac{D_x}{2F_0}\frac{\partial^2F_0}{\partial x^2}.$$

All matrices are diagonal except Γ. Note that the elements of $\tilde{\Omega}$ now depend on $\mathbf{k}$, since they include diffusion terms, while in Mattor and Parker's work they were the same for different $\mathbf{k}$. Inverting the evolution operator gives

$$\vec{f}=-(v_{\|}-\mathcal{W})^{-1}\mathcal{K}_{\|}^{-1}\Omega_*(v_{\|})F_0. \tag{10}$$

Thus $\mathcal{W}\equiv\mathcal{K}_{\|}^{-1}[\tilde{\Omega}-\Gamma]$ is nonlinear phase velocity matrix,

$$\int dv_{\|}\left(-\left[\frac{\tau}{c_s^2}\right]^5v_{\|}^5+10\left[\frac{\tau}{c_s^2}\right]^4v_{\|}^3-15\left[\frac{\tau}{c_s^2}\right]^3v_{\|}\right)\vec{f}$$
$$=-\int dv_{\|}\frac{\Omega_*(v_{\|})\mathcal{K}_{\|}^{-1}}{v_{\|}-\mathcal{W}}\partial_{v_{\|}}^5F_0$$
$$=-\partial_\alpha^5\left[\int dv_{\|}\frac{F_0}{v_{\|}-\mathcal{W}-\alpha}\right]_{\alpha=0}\Omega_*(\mathcal{W})\mathcal{K}_{\|}^{-1}, \tag{11}$$

$$\int dv_{\|}\left(\left[\frac{\tau}{c_s^2}\right]^4v_{\|}^4-6\left[\frac{\tau}{c_s^2}\right]^3v_{\|}^2+3\left[\frac{\tau}{c_s^2}\right]^2\right)\vec{f}$$
$$=-\int dv_{\|}\frac{\Omega_*(v_{\|})\mathcal{K}_{\|}^{-1}}{v_{\|}-\mathcal{W}}\partial_{v_{\|}}^4F_0$$
$$=-\partial_\alpha^4\left[\int dv_{\|}\frac{F_0}{v_{\|}-\mathcal{W}-\alpha}\right]_{\alpha=0}\Omega_*(\mathcal{W})\mathcal{K}_{\|}^{-1}. \tag{12}$$

We have applied the identity

$$\partial_v^m \frac{v^n}{v-\mathcal{W}} = (-\partial_\alpha)^m \frac{\mathcal{W}^n}{v-\alpha-\mathcal{W}}\bigg|_{\alpha=0} \quad (m\geqslant n).$$

Then using the definitions of fluid moments we can express the term that we are interested in,

$$\frac{\int dv_\parallel v_\parallel^5 \vec{f}}{n_0 c_s^5} = -\frac{Z_0^{[5]}(\mathcal{W}\sqrt{\tau/2})}{Z_0^{[4]}(\mathcal{W}\sqrt{\tau/2})}\sqrt{\frac{\tau}{2}}\left(\frac{1}{\tau}\vec{J}_\parallel - \frac{6}{\tau^2}\vec{T}_\parallel - \frac{3}{\tau^3}\vec{n}\right) + \frac{10}{\tau}\vec{Q}_\parallel - \frac{15}{\tau^2}\vec{V}_\parallel, \tag{13}$$

where Z_0 is the analytically continued plasma dispersion function

$$Z_0(\zeta) \equiv \frac{1}{\sqrt{\pi}}\int_{C_L} dx\, \frac{e^{-x^2}}{x-\zeta}.$$

The superscripts [5] and [4] denote derivatives evaluated with α like in Eqs. (11) and (12). The obtained form of highest moment (13) emphasizes the fact that it can depend on all lower moments, while the closure in the third moment (in Ref. 1) shows only the dependence of two of them. The main influences of diffusion on the closure procedure appear in the changing of the order of velocity moment on which we may close hierarchy, and changing the structure of nonlinear phase velocity matrix $\mathcal{W}$ by adding a permanent imaginary part to frequencies.

IV. THREE-MODE ITG SYSTEM

We apply the developed approach for a particular case of three-mode ITG system with diffusion, specified in the following. The equilibrium has a straight tilted magnetic field $\mathbf{B}=B(\hat{\mathbf{x}}+\Theta\hat{\mathbf{y}})$. The spectrum is truncated at three modes with $\mathbf{k}=(k_x,k_y)$, $\mathbf{k}=(k_x,-k_y)$, $(2k_x,0)$, denoted $+$, $-$ and 0, respectively. The perturbation in $\tilde{f}$ and $\tilde{\phi}$ are periodic in y, vanish at $x=0,L_x$, and have $k_z=0$ so $k_\parallel=\Theta k_y$. It gives us the following important relations:

$$\tilde{f}_- = -\tilde{f}_+^*, \qquad \tilde{\phi}_- = -\tilde{\phi}_+^*, \qquad \mathrm{Re}(\tilde{f}_0)=\mathrm{Re}(\omega_0)=0. \tag{14}$$

We also take $\tilde{\phi}_0\equiv 0$.

According to this specification of the ITG system, we need to restrict ourselves by considering only diffusion (no viscosity) and neglecting in fluid equations (4)–(8) terms produced by $\partial^2 F_0/\partial x^2$. This could be easily proven, since in most cases these terms are of order 10^{-5}–10^{-6}.

The kinetic equation (2) for this system in vector form is

$$-i(\tilde{\Omega}-v_\parallel\mathcal{K}_\parallel)\vec{f} - \tilde{c}\kappa\begin{pmatrix}\tilde{n}_+\\ -\tilde{n}_-\end{pmatrix}\tilde{f}_0 = -i\Omega_*(v_\parallel)F_0, \tag{15}$$

$$-i(\omega_0+2iD_xk_x^2)\tilde{f}_0 - \tilde{c}\kappa(\tilde{n}_- - \tilde{n}_+)\vec{f}=0, \tag{16}$$

where $\vec{f}$ is column vector of $\tilde{f}_\pm$, and $\kappa=2k_xk_y$. Note, that here frequencies are normalized by $|\omega_{*e}|$, velocity by c_s, k by ρ_s^{-1} and distribution functions by n_0/c_s, $\mathcal{K}_\parallel$ has $\tilde{c}k_\parallel$ on its diagonal.

Substituting $\tilde{f}_0$ from Eq. (16) into Eq. (15) gives us the explicit form of matrix $\mathcal{W}$ in Eq. (10)

$$\mathcal{W}\equiv\begin{pmatrix} w_{11} & w_{12}\\ w_{21} & w_{22}\end{pmatrix} = \begin{pmatrix} v_p+is_0|u| & u\\ u^* & v_p^*-is_0|u|\end{pmatrix}. \tag{17}$$

Here

$$v_p=\frac{\omega_+ + i(D_xk_x^2+D_yk_y^2)}{k_\parallel\tilde{c}},$$

$$u=-\frac{\tilde{c}\kappa^2}{k_\parallel(\omega_0+2iD_xk_x^2)},$$

$$s_0=\mathrm{sign}[\mathrm{Im}(\omega_0+2iD_xk_x^2)].$$

Frequencies we define in eikonal fashion, $-i\omega_+=\partial_t\ln\tilde{n}_+$, and $-i\omega_0=\partial_t\ln\tilde{J}_{\parallel 0}$.

For handling with the matrix function $\hat{B}(\mathcal{W}) \equiv -Z_0^{[5]}(\mathcal{W}\sqrt{\tau/2})/Z_0^{[4]}(\mathcal{W}\sqrt{\tau/2})$ we use an expression

$$\hat{B}(\mathcal{W})\hat{A}=\sum_j \hat{B}(w_j)A_j\vec{\xi}_j,$$

where $\{\vec{\xi}_j\}$ is the complete set of eigenvectors of $\mathcal{W}$, such that $\mathcal{W}\vec{\xi}_j=w_j\vec{\xi}_j$, $\vec{A}=\Sigma_jA_j\vec{\xi}_j$.

There are two eigenvalues of $\mathcal{W}$,

$$w_\pm=\frac{w_{11}+w_{22}}{2}\pm s_w\sqrt{\left(\frac{w_{11}-w_{22}}{2}\right)^2+w_{12}w_{21}},$$

where sign $s_w=\pm 1$ should be chosen from the continuity arguments. Then the $+$ component of the product $\hat{B}\vec{A}$ can be expressed as

$$[\hat{B}(\mathcal{W})\vec{A}]_+=\frac{\hat{B}_+-\hat{B}_-}{w_+-w_-}\left[\frac{w_{11}-w_{22}}{2}A_+-w_{12}A_-\right]+\frac{1}{2}(\hat{B}_+-\hat{B}_-)A_+,$$

where

$$\vec{A}=\begin{pmatrix}A_+\\ A_-\end{pmatrix},\quad \hat{B}_\pm\equiv\hat{B}(w_\pm).$$

Now we have everything we need to construct the closure term (13),

$$\frac{\int dv_\parallel v_\parallel^5\tilde{f}_+}{n_0c_s^5}=\sqrt{\frac{\tau}{2}}[\hat{B}(\mathcal{W})\vec{A}]_+ + \frac{10}{\tau}\vec{Q}_\parallel - \frac{15}{\tau^2}\vec{V}_\parallel, \tag{18}$$

here

$$\vec{A}=\frac{1}{\tau}\vec{J}_\parallel-\frac{6}{\tau^2}\vec{T}_\parallel-\frac{3}{\tau^3}\vec{n}.$$

We have solved the system of fluid equations (4)–(8) with the closure term of form (18) numerically. The time

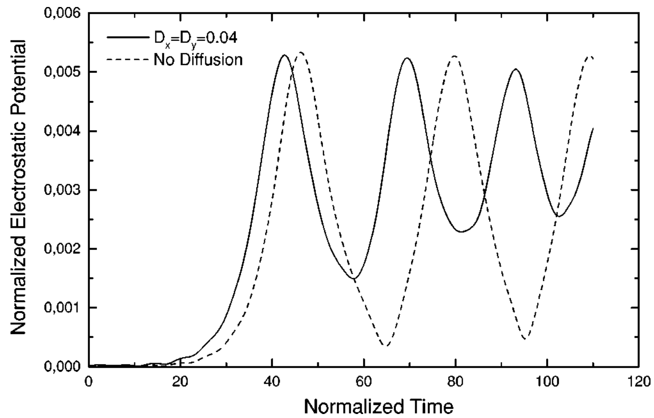

FIG. 1. Time evolution of $|\tilde{n}_+|$. Here $k_x = k_y = 0.3$, $\Theta = 0.001$, $\eta_i = 3$.

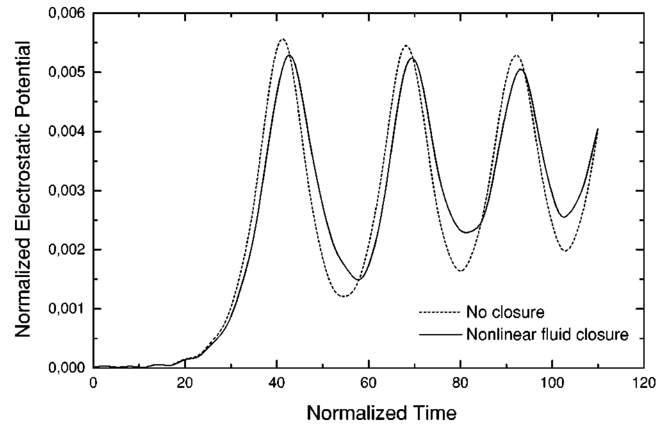

FIG. 2. Time evolution of $|\tilde{n}_+|$ for the system with nonlinear fluid closure and without closure term. $k_x = k_y = 0.3$, $\Theta = 0.001$, $\eta_i = 3$.

evolution of $\tilde{n}_+$ (i.e., electrostatic potential) is shown in Fig. 1. The bounces arise from resonant ions orbiting in the potential well of $\tilde{\phi}_+$ (see Ref. 7) and demonstrate the retaining of particle trapping by nonlinear fluid closure. The suppressing of the trapping amplitude shows that we lose time reversibility of the kinetic equation by introducing diffusion.

V. CONCLUSIONS

We have studied a simple three-wave system of the same type as studied by Mattor and Parker in order to improve our understanding of fluid closures in general, and of the different results obtained by different fluid closures in the Cyclone in particular.

The present system is somewhat limited in the sense that the fluid growth rate and the closure are both associated with parallel ion motion and thus tied together. In general, the toroidal drive of the drift wave dominates, while the closure remains mainly associated with the parallel ion motion.[8]

The simple Hammett–Perkins closure[9] is equivalent to adding a diffusive heat flux. It was found by Mattor and Parker to give a saturation level just above the maxima of the oscillations in our system. We have also added diffusion as a way of including interaction with a background turbulence. It enters in a way similar to the closure term in the Hammett–Perkins model and makes the system approach an asymptotic stationary state for large times.

The system is very similar to that given in Ref. 10 where a nonlinearly unstable three wave system was stabilized by a nonlinear frequency shift. For that system, which was more explicit, the asymptotic level could be given analytically as a function of the coefficients of the nonlinear frequency shift and the dampings (diffusion terms).

It is also interesting to note that for the nonlinearly instable three-wave system mentioned previously, an imaginary part of the nonlinear frequency shift gave a much more rapid approach to the asymptotic state, which also appeared to be located at a lower level. This is similar to the nonlinear kinetic Cyclone results for the turbulent case.[2]

We also note that the system without closure (Fig. 2) also has a very similar behavior. The rather small difference seems to be due to the close relation between linear growth and closure mentioned previously.

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