Electron continuity equation

Normalized continuity equation

$$\frac{\partial \delta n_e}{\partial t} + B_0 \mathbf{b}_0 \cdot \nabla \left(\frac{n_0 \delta u_{\parallel e}}{B_0} \right) + \mathbf{v}_E \cdot \nabla n_0 + \mathbf{v}_{E0} \cdot \nabla \delta n_e - n_0 \mathbf{v}_* \cdot \frac{\nabla B_0}{B_0} - 2 \left(n_0 \mathbf{v}_E + \delta n_e \mathbf{v}_{E0} \right) \cdot \frac{\nabla B_0}{B_0} - \frac{1}{B_0^2} \nabla \times \mathbf{B}_0 \cdot \nabla \delta P_\parallel = 0 \quad (1)$$

where

$$\mathbf{v}_* = -\frac{1}{n_0 B_0} \mathbf{b}_0 \times \nabla \left(\delta P_\parallel + \delta P_\perp \right)$$

$$\delta P_\perp = \int d\mathbf{v} \mu B_0 \delta f = n_0 \phi_{\text{eff}} + \frac{\partial P_0}{\partial \psi} \delta \psi \quad (2)$$

$$\delta P_\parallel = \int d\mathbf{v} m v_\parallel^2 \delta f = n_0 \phi_{\text{eff}} + \frac{\partial P_0}{\partial \psi} \delta \psi \quad (3)$$

$$P_0 = n_0 T_e$$

Using

$$\mathbf{B}_0 = I \nabla \theta + g \nabla \zeta, \quad (4)$$

$$J^{-1} = \nabla \psi \cdot \nabla \theta \times \nabla \zeta = \frac{B_0^2}{gq + I} \quad (5)$$

we get the $\mathbf{v}_*$-term

$$n_0 \mathbf{v}_* \cdot \frac{\nabla B_0}{B_0} = -\frac{2}{J B_0^3} \left[g \left(\frac{\partial (n_0 \phi_{\text{eff}})}{\partial \psi} + \frac{\partial^2 P_0}{\partial \psi^2} \delta \psi + \frac{\partial P_0}{\partial \psi} \frac{\partial \delta \psi}{\partial \psi} \right) \frac{\partial B_0}{\partial \theta} - g \left(\frac{\partial (n_0 \phi_{\text{eff}})}{\partial \theta} + \frac{\partial P_0}{\partial \psi} \frac{\partial \delta \psi}{\partial \theta} \right) \frac{\partial B_0}{\partial \psi} + I \left(\frac{\partial (n_0 \phi_{\text{eff}})}{\partial \zeta} + \frac{\partial P_0}{\partial \psi} \frac{\partial \delta \psi}{\partial \zeta} \right) \frac{\partial B_0}{\partial \psi} \right] \quad (6)$$

and $\mathbf{v}_E$-term

$$n_0 \mathbf{v}_E \cdot \frac{\nabla B_0}{B_0} = \frac{n_0}{J B_0^3} \left[I \frac{\partial \phi}{\partial \zeta} \frac{\partial B_0}{\partial \psi} + g \left(\frac{\partial \phi}{\partial \psi} \frac{\partial B_0}{\partial \theta} - \frac{\partial \phi}{\partial \theta} \frac{\partial B_0}{\partial \psi} \right) \right]. \quad (7)$$

The last term in Eq. (1) vanishes in curl-free magnetic field approximation. In general case, however, it can be written as

$$\frac{1}{B_0^2} \nabla \times \mathbf{B}_0 \cdot \nabla \delta P_\parallel = \frac{1}{J B_0^2} \left[\frac{\partial I}{\partial \psi} \left(\frac{\partial (n_0 \phi_{\text{eff}})}{\partial \zeta} + \frac{\partial P_0}{\partial \psi} \frac{\partial \delta \psi}{\partial \zeta} \right) - \frac{\partial g}{\partial \psi} \left(\frac{\partial (n_0 \phi_{\text{eff}})}{\partial \theta} + \frac{\partial P_0}{\partial \psi} \frac{\partial \delta \psi}{\partial \theta} \right) \right] \quad (8)$$

If the background radial electric field is present

$$\mathbf{v}_{E0} \cdot \nabla \delta n_e = \frac{1}{JB_0^2} \left[g \frac{\partial \phi_0}{\partial \psi} \frac{\partial \delta n_e}{\partial \theta} - I \frac{\partial \phi_0}{\partial \psi} \frac{\partial \delta n_e}{\partial \zeta} \right] \tag{9}$$

$$\delta n_e \mathbf{v}_{E0} \cdot \frac{\nabla B_0}{B_0} = \frac{\delta n_e}{JB_0^3} g \frac{\partial \phi_0}{\partial \psi} \frac{\partial B_0}{\partial \theta}. \tag{10}$$