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Alfvén waves: a journey between space and fusion plasmas

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Abstract
Alfvén waves discovered by Hannes Alfvén (1942 *Nature* **150** 405) are fundamental electromagnetic oscillations in magnetized plasmas existing in the nature and laboratories. Alfvén waves play important roles in the heating, stability and transport of plasmas. The anisotropic nearly incompressible shear Alfvén wave is particularly interesting since, in realistic non-uniform plasmas, its wave spectra consist of both the regular discrete and the singular continuous components. In this Alfvén lecture, I will discuss these spectral properties and examine their significant linear and nonlinear physics implications. These discussions will be based on perspectives from my own research in both space and laboratory fusion plasmas, and will demonstrate the positive feedback and cross-fertilization between these two important sub-disciplines of plasma physics research. Some open issues of nonlinear Alfvén wave physics in burning fusion as well as magnetospheric space plasmas will also be explored.

1. Introduction

In 1942, Hannes Alfvén discovered that, in the presence of a finite magnetic field $\boldsymbol{B}_0$, electromagnetic waves can propagate inside a conducting fluid such as a plasma [1]. These hydromagnetic waves, called Alfvén waves in honour of him, have a phase velocity given by the Alfvén speed $V_A = B_0/(4\pi n_i m_i)^{1/2}$ with $n_0 m_i$ being the mass density. Since Alfvén waves carry electric and magnetic fields, and, hence, wave energy and momentum, they have been found to play important roles in the dynamics of magnetized plasmas existing in both the nature and laboratory environments.

Within the ideal magnetohydrodynamic (MHD) description, there exist two types of Alfvén waves. One is the isotropic compressional Alfvén wave (CAW), and the other is the anisotropic shear Alfvén wave (SAW). Since SAW is nearly incompressible, it can be more readily excited by collective effects than CAW. SAW, therefore, has received much more attention and will be the focus of this lecture.

My own research 'journey' of SAW physics began in 1972 when I joined the Hasegawa-Lanzerotti group at AT&T Bell Laboratories at Murray Hill, New Jersey, as a postdoctoral researcher. For reasons unjustifiable nowadays, Bell Laboratories then had strong research efforts in basic plasma and space physics; including an array of ground magnetometer stations maintained by Lanzerotti. I was asked by Akira Hasegawa to provide theoretical explanations for the observed geomagnetic oscillations. For me, the conceptual breakthrough occurred when I read Grad's review paper in *Physics Today* [2]; where he commented on the concepts of phase mixing and continuum spectrum and remarked that 'phase mixing with collisionless damping is not restricted to kinetic models but is also found in macroscopic theory of Alfvén waves...'. After my postdoctoral years, I consider myself very fortunate that I have been able to continue and further extend my 'journey' of Alfvén wave physics research in space and fusion plasmas in a dynamic scientific research institution, Princeton Plasma Physics Laboratory (see acknowledgment), during a period of extraordinary intellectual activities.

In the following lecture, I would like to humbly sketch my own experiences and understandings of Alfvén wave physics in space and fusion plasmas, and demonstrate, hopefully, the positive and fruitful cross-fertilizations between these two important areas of plasma physics research. Along this lecture, I will also comment on some of what I consider to be significant outstanding issues. A word of caution is needed here. As the subject of Alfvén wave physics is broad and my own knowledge is limited, this lecture is never intended to be a comprehensive review on this important subject and many important topics will, thus, not be addressed here.

2. Geomagnetic pulsations, continuous spectrum and kinetic Alfvén waves (KAWs)

Geomagnetic pulsations are oscillations in the Earth's magnetic fields with periods typically in the range of $\mathcal{O}(10\text{–}10^3)$ s [3]. It has long been theoretically recognized that these pulsations correspond to hydromagnetic Alfvén waves in the Earth's magnetospheric plasmas [4]. In the early 1970s, ground magnetometer observations have consistently shown interesting but puzzling features of both the wave amplitude and polarization on the latitude and morning–afternoon asymmetry [3, 5]. Comprehensive theoretical studies [6–8] have led to the appreciation that, with non-uniformities perpendicular to the confining magnetic field, the transverse SAW exhibits the dynamics of continuous spectrum along with resonant absorption [9], phase mixing [2] and mode conversion to KAW [10, 11].

Let us illustrate the above linear wave properties by considering a plasma immersed in a uniform magnetic field $\boldsymbol{B}_0 = B_0\hat{z}$ but with a non-uniform density $n_0 = n_0(x)$ for $-\infty < x < \infty$. For simplicity, let us assume that the plasma is confined by two conducting plates at $z = 0$ and L. Furthermore, for now, we adopt the ideal MHD description, and take the plasma to be cold. It is then straightforward to derive the following set of coupled equations [12]

$$\mathcal{D}_A \delta\boldsymbol{\zeta}_\perp = \boldsymbol{\nabla}_\perp(\delta B_\parallel/B_0), \tag{1}$$

$$\delta B_\parallel/B_0 = \boldsymbol{\nabla}\cdot\delta\boldsymbol{\zeta}_\perp; \tag{2}$$

where

$$\mathcal{D}_A = \partial^2/\partial z^2 + \omega^2/V_A^2(x) \tag{3}$$

is the local SAW operator, $V_A(x) = B_0/[4\pi m_i n_0(x)]^{1/2}$ is the local Alfvén speed, $\delta\boldsymbol{\zeta}_\perp$ is the fluid displacement vector and $\delta B_\parallel = \delta\boldsymbol{B}\cdot\boldsymbol{B}_0/B_0$ is the compressional component of the perturbed magnetic field, $\boldsymbol{\nabla}_\perp = \hat{\boldsymbol{x}}\partial/\partial x + \hat{\boldsymbol{y}}\mathrm{i}k_y$ and $\exp(-\mathrm{i}\omega t + \mathrm{i}k_y y)$ dependence is assumed. (1) and (2) can be combined in an operator form

$$\left[\frac{\partial}{\partial x}\frac{\mathcal{D}_A}{k_y^2 - \mathcal{D}_A}\frac{\partial}{\partial x} - \mathcal{D}_A\right]\delta\zeta_x(x,z) = 0. \tag{4}$$

Note in uniform plasmas, $\partial\mathcal{D}_A/\partial x = 0$; (4) reduces to $\mathcal{D}_A\delta\zeta_x = 0$ and $[\partial^2/\partial x^2 - k_y^2 + \mathcal{D}_A]\delta\zeta_x = 0$; i.e. uncoupled SAW and CAW, respectively. With perpendicular non-uniformity, $\partial\mathcal{D}_A/\partial_x \neq 0$, SAW and CAW are, thus, coupled together.

Taking further $\delta\zeta_x(x,z) = \delta\hat{\zeta}_{xn}(x)\sin(k_n z)$ with $k_n = n\pi/L$, n being positive integers, (4) becomes [8]

$$\left[\frac{\mathrm{d}}{\mathrm{d}x}\frac{\varepsilon_{An}}{k_y^2 - \varepsilon_{An}}\frac{\mathrm{d}}{\mathrm{d}x} - \varepsilon_{An}\right]\delta\hat{\zeta}_{xn}(x) = 0, \tag{5}$$

where $\varepsilon_{An} = \omega^2/V_A^2(x) - k_n^2$. (5) exhibits singular behavior when $\varepsilon_{An} = 0$ or $\omega^2 = k_n^2 V_A^2(x) \equiv \omega_{An}^2(x)$. Since $\omega_{An}^2(x)$ varies continuously in x due to $n_0(x)$, $\omega_{An}^2(x)$ constitutes a continuous spectrum or a SAW continuum.

Given $\omega = \omega_0$ being the CAW driving frequency, $x = x_{nr}$ where $\omega_{An}^2(x_{nr}) = \omega_0^2$ then corresponds to the Alfvén resonance layer. Near $x = x_{nr}$, we can readily solve (5) and obtain

$$\delta\hat{\zeta}_{xn} \simeq (c_n/\varepsilon'_{An})\ln(x - x_{nr}), \tag{6}$$

and

$$\delta\hat{\zeta}_{yn} \simeq \mathrm{i}(c_n/k_y\varepsilon'_{An})/(x - x_{nr}); \tag{7}$$

where $\varepsilon'_{An} = \mathrm{d}\varepsilon_{An}/\mathrm{d}x$ at $x = x_{nr}$, and c_n is related to the CAW amplitude at $x = x_{nr}$; i.e.

$$c_n = k_y^2\delta\hat{B}_\parallel/B_0. \tag{8}$$

The above results extended to include dissipations can then successfully explain the observed features of geomagnetic pulsations [6]. In the space physics community, this model is referred to as the field-line-resonance theory of magnetic pulsations. Note that the existence of SAW continuum in general dipole-like magnetic field geometries has also been demonstrated [13].

That $\delta\hat{\zeta}_\perp$ becomes singular at $x = x_{nr}$ physically corresponds to piling up of wave energy carried by CAW when the driving frequency matches the local SAW frequency. The corresponding resonant energy absorption rate per unit length in y can be calculated and is given by [9]

$$\mathrm{d}W/\mathrm{d}t = (\omega_0/8)L|k_y\delta\hat{B}_\parallel|^2/|\varepsilon'_{An}|. \tag{9}$$

Note that since this absorption rate is, in essence, the piling-up rate, it is 'collisionless'. Within the ideal MHD model, this Alfvén resonance heating is local and has been suggested for both laboratory plasma heating experiments [9] and as a possible mechanism responsible for heating the solar corona [14].

Since geomagnetic pulsations are often excited by external pulse-like perturbations (e.g. solar-wind pressure pulses on the dayside magnetosphere), it is also instructive to examine the SAW oscillations in terms of time-asymptotic responses. Noting, a posterior that, as $t \to \infty$, $|\partial^2/\partial x^2| \gg |k_y^2| \gg |\varepsilon_{An}|$, (5) becomes, restoring the time dependence,

$$\frac{\partial}{\partial x}\left[\frac{\partial^2}{\partial t^2} + \omega_{An}^2(x)\right]\frac{\partial}{\partial x}\delta\hat{\zeta}_{xn}(x,t) = 0. \tag{10}$$

(10) readily gives

$$\delta\hat{\zeta}_{yn}(x,t) = (\mathrm{i}/k_y)(\partial\delta\hat{\zeta}_{xn}/\partial x) = (\mathrm{i}/k_y)c_n \exp[\pm\mathrm{i}\omega_{\mathrm{A}n}(x)t] \tag{11}$$

and

$$\delta\hat{\zeta}_{xn}(x,t) \simeq \mp\mathrm{i}\frac{c_n}{\omega'_{\mathrm{A}n}(x)}\frac{1}{t}\exp[\pm\mathrm{i}\omega_{\mathrm{A}n}(x)t]. \tag{12}$$

(11) clearly manifests that $\delta\hat{\zeta}_{yn}$; i.e. transverse perturbations orthogonal to the inhomogeneity direction (x), exhibits undamped oscillation at the local SAW oscillation. $\delta\hat{\zeta}_{xn}$, meanwhile, exhibits $(1/t)$ temporal decay due to phase mixing of SAW continuum with finite $\omega'_{\mathrm{A}n} = \mathrm{d}\omega_{\mathrm{A}n}/\mathrm{d}x$. In terms of the Earth's magnetosphere, $\delta\zeta_y$ and $\delta\zeta_x$ correspond, respectively, to East–West and radial magnetic perturbations. Measurements from AMPTE/CCE satellites have indeed observed transverse magnetic perturbations exhibiting clear bands of SAW continuous spectra with the East–West component having stronger intensity than the radial component [15]. Note also that (11) and (12) define an effective wave number, $k_x(t) = |\omega'_{\mathrm{A}n}t|$. Thus, consistent with the steady-state solutions, (6) and (7), the perturbations will evolve into singular structures time asymptotically.

That singular structures develop time asymptotically or at the steady state, of course, suggests that we need to include kinetic effects associated with microscopic perpendicular scale lengths such as ion Larmor radii, which has been neglected in the MHD approximation. The resultant kinetic SAW (KAW) equation then becomes [10, 11]

$$\nabla_\perp \cdot (\varepsilon_{\mathrm{KA}n}\nabla_\perp\delta\hat{\zeta}_{xn}) = 0; \tag{13}$$

where

$$\varepsilon_{\mathrm{KA}n} = k_n^2\rho_k^2\nabla_\perp^2 + \varepsilon_{\mathrm{A}n}, \tag{14}$$

$$\rho_k^2 = \rho_\mathrm{i}^2\left[\frac{3}{4}(1-\mathrm{i}\delta_\mathrm{i}) + \frac{T_\mathrm{e}}{T_\mathrm{i}}(1-\mathrm{i}\delta_\mathrm{e})\right]; \tag{15}$$

δ_i and δ_e are, respectively, ion and electron dissipations, ρ_i is the ion Larmor radius and $1 \gg \beta_\mathrm{e} \gg m_\mathrm{e}/m_\mathrm{i}$ is assumed[3]. Here, $\beta_\mathrm{e} = 8\pi P_\mathrm{e}/B_0^2$ and P_e is the electron pressure. (14) also assumes $|\nabla_\perp^2\rho_k^2| < 1$. Away from the resonance layer at $x = x_{n\mathrm{r}}$, we may adopt the WKB approximation for the KAW dispersion relation

$$\omega^2 \simeq \omega_{\mathrm{A}n}^2(x)[1 + k_\perp^2(x)\rho_k^2]. \tag{16}$$

Near the resonance layer, $\varepsilon_{\mathrm{A}n} \simeq \varepsilon'_{\mathrm{A}n}(x - x_{n\mathrm{r}})$, (13) can be solved for the linear mode conversion process in terms of Airy functions along with the appropriate boundary conditions [11]. It is instructive to examine the following solutions valid sufficiently away from the resonance layer:

$$\delta\hat{\zeta}_{yn} = \frac{\mathrm{i}c_n}{k_y k_n^2}\left\{\frac{1}{\kappa s} - \frac{\sqrt{\pi}}{(\kappa\rho_k)^{2/3}}\left(\frac{\Delta}{s}\right)^{1/4}\exp\left[\mathrm{i}\left(\frac{2}{3}\left(\frac{s}{\Delta}\right)^{3/2} + \frac{1}{4}\right)\right]\right\}, \qquad \text{for } \omega^2 > \omega_{\mathrm{A}n}^2; \tag{17}$$

and

$$\delta\hat{\zeta}_{yn} = \frac{\mathrm{i}c_n}{k_y k_n^2}\frac{1}{\kappa s}, \qquad \text{for } \omega^2 < \omega_{\mathrm{A}n}^2; \tag{18}$$

where $\varepsilon_{\mathrm{A}n} \simeq \varepsilon'_{\mathrm{A}n}s$, $s = x - x_{nr}$, $\kappa = \varepsilon'_{\mathrm{A}n}/k_n^2$ and $\Delta = (\rho_k^2/\kappa)^{1/3}$. The first terms in (17) and (18) correspond to the ideal MHD solutions, (7). The second term in (17), meanwhile, corresponds to the mode-converted KAW propagating outward in the $\omega^2 > \omega_{\mathrm{A}n}^2(x)$ domain.

[3] For $\beta_\mathrm{e} \ll m_\mathrm{e}/m_\mathrm{i}$, we would have the so-called inertial Alfvén wave or a cold 'surface wave.' [16].

Physically, KAW removes the ideal MHD singularly since the wave energy can no longer pile indefinitely due to the finite perpendicular group velocity of KAW;

$$\boldsymbol{v}_{g\perp} = \frac{\partial \omega_{KAW}}{\partial \boldsymbol{k}_\perp} = \boldsymbol{k}_\perp \frac{\omega_{An}^2}{\omega} \rho_k^2. \tag{19}$$

More significantly, in terms of wave–particle interactions, KAW has a finite $\delta E_\parallel$;

$$|\delta E_\parallel / \delta E_\perp| \simeq k_\parallel k_\perp \rho_s^2. \tag{20}$$

Here, $k_\parallel = k_n$ and $\rho_s = (T_e/m_i)^{1/2}/\Omega_i$. KAW can thus lead to acceleration and heating of charged particles along $\boldsymbol{B}$, breaking the constants of motion, and, consequently, transports across $\boldsymbol{B}$. These features have found applications in auroral physics [17], solar corona heating [14] and transports at dayside magnetopause [18–20].

It is worthwhile to remark that this SAW–KAW model could serve as a paradigm for other cases with similar continuum and resonance structures; e.g. the geodesic-acoustic mode (GAM) [21] and the mode conversion to kinetic geodesic-acoustic mode (KGAM) [22].

3. SAWs in toroidal plasmas

In the preceding section, we have demonstrated the existence of SAW continuum due to the perpendicular (radial in the case of tokamaks) inhomogeneities. For tokamaks, the asymmetry in the poloidal direction introduces an additional asymmetry for SAW along $\boldsymbol{B}$. While it is recognized that this parallel asymmetry along field lines on a magnetic surface results in gaps in the SAW continuum [23], systematic theoretical studies [24] become feasible only after the development of the ballooning-mode formalism for modes with high toroidal mode numbers [25–27].

Let us consider, for now, an axisymmetric tokamak with a large aspect ratio $\varepsilon = a/R \ll 1$ and shifted circular magnetic surfaces. Let r, χ and ζ be, respectively, the radial, poloidal-angle and toroidal-angle coordinates. The confining magnetic field is given by

$$\boldsymbol{B} = B_\zeta(r,\chi)\hat{\zeta} + B_p(r,\chi)\hat{\chi},$$

with B_ζ and B_p being the toroidal and poloidal magnetic fields, respectively.

Expressing the perturbation as

$$\delta\phi(r,\chi,\zeta,t) = \exp(-i\omega t + in\zeta)\sum_m \delta\phi(r,m)\exp(-im\chi) + \text{c.c.},$$

we shall adopt, for $n \gg 1$, the lowest translational-invariance ordering; i.e.

$$\delta\phi(r,m) = \Phi(nq - m),$$

with $q = rB_\zeta/RB_p$ being the safety factor. Let $\hat{\Phi}(\eta)$ be the Fourier conjugate of $\Phi(nq-m)$, the SAW equation in the cold ideal MHD limit then becomes [12, 24]

$$\left[\frac{d}{d\eta} f \frac{d}{d\eta} + \Omega^2(1 + 2\varepsilon_0\cos\eta) f\right]\hat{\Phi}(\eta) = 0, \qquad -\infty < \eta < \infty; \tag{21}$$

where $f(\eta) = 1 + \hat{s}^2\eta^2$, $\hat{s} = rq'/q$ denotes the magnetic shear, $\Omega^2 = (\omega/\omega_A)^2$, $\omega_A = V_A/qR$, $\varepsilon_0 = r/R + \Delta_s'$ and Δ_s is the Shafranov shift.

Letting $\hat{\Psi}(\eta) = f^{1/2}\hat{\Phi}(\eta)$, (21) can be cast into the following Schrödinger equation form

$$\left[\frac{d^2}{d\eta^2} + V(\eta)\right]\hat{\Psi}(\eta) = 0; \tag{22}$$

where

$$V(\eta) = \Omega^2(1 + 2\varepsilon_0 \cos\eta) - (\hat{s}/f)^2. \tag{23}$$

For the SAW continuum, which corresponds to singular solutions in $\hat{\Phi}(nq - m)$, we shall examine (22) in the corresponding $|\eta| \to \infty$ limit; i.e.

$$V(\eta) \to V_c(\eta) = \Omega^2(1 + 2\varepsilon_0 \cos\eta). \tag{24}$$

(22) with $V_c(\eta)$ is the standard Mathieu's equation; i.e. a wave equation in a periodic structure due to the $2\varepsilon_0 \cos\eta$ term. For $\varepsilon_0 = 0$ the SAW continuum is given by $0 \leqslant \Omega^2 < \infty$ corresponding to $\omega^2 = (nq-m)^2\omega_A^2$. Finite $\varepsilon_0 \neq 0$ then introduces Bloch reflections when the wavelengths match the integral multiples of the 'lattice' period. More precisely, $\hat{\Psi}_c(\eta)$ becomes non-propagating in narrow frequency gaps, $(\Delta\Omega)_j \sim \mathcal{O}(\varepsilon_0^j)$, centered about the characteristic frequencies at $\Omega_j^2 = (j/2)^2$ for $j = 1, 2, \ldots$. Since propagating solutions correspond to SAW continuum, 'lattice' symmetry, thus, introduces gaps to the SAW continuum. We can, of course, readily understand these frequency 'gaps' from the finite couplings between the mth and $(m + j)$th poloidal harmonics due to the $\varepsilon_0 \neq 0$ finite toroidicity or poloidal asymmetries.

The $(\hat{s}/f)^2$ term in (23), however, breaks the 'lattice' symmetry at $|\eta| \sim \mathcal{O}(1)$. One may view it as an effective 'defect' in an otherwise purely periodic 'lattice'. For Ω^2 residing inside the gaps, bound states which decay exponentially for $|\eta| \gg 1$ could then be formed due to the finite 'defect'-induced potential well around $|\eta| \sim \mathcal{O}(1)$. For SAW in tokamaks, these discrete marginally stable 'defect'-induced bound states are called toroidicity-induced shear Alfvén eigenmodes or TAE [24].

To be more specific, we shall assume $\varepsilon_0, \hat{s} \ll 1$ and derive the TAE dispersion relation via the two-scale lengths $[\eta_0 \sim \mathcal{O}(1), \eta_1 \sim \mathcal{O}(1/\hat{s})]$ expansions [28]. Considering the $j = 1$ widest (toroidal Alfvén) gap, we let

$$\hat{\Psi}(\eta) = A(\eta_1)\cos(\eta_0/2) + B(\eta_1)\sin(\eta_0/2). \tag{25}$$

(22) then becomes, neglecting $\mathrm{d}^2A/\mathrm{d}\eta_1^2$ and $\mathrm{d}^2B/\mathrm{d}\eta_1^2$ terms,

$$\mathrm{d}A/\mathrm{d}\eta_1 = (\Gamma_- - \hat{s}^2/f^2)B(\eta_1) \tag{26}$$

and

$$\mathrm{d}B/\mathrm{d}\eta_1 = -(\Gamma_+ - \hat{s}^2/f^2)A(\eta_1); \tag{27}$$

where

$$\Gamma_{+,-} = \Omega^2 - \Omega_{L,U}^2, \tag{28}$$

and

$$\Omega_{L,U}^2 = [4(1 \pm \varepsilon_0)]^{-1}. \tag{29}$$

(26) and (27) can be solved via asymptotic matching. For the $|\eta_1| \gg 1$ external 'singular' region, we have, considering only the even mode,

$$(A, B)_E = (A_O, B_O \exp)(-\hat{\gamma}\eta_1), \tag{30}$$

where $\hat{\gamma} = (-\Gamma_-\Gamma_+)^{1/2}$ and $(B_O/A_O) = (-\Gamma_+/\Gamma_-)^{1/2}$. For the internal 'regular' $|\eta_1| \sim 1$ region, the 'defect'-induced potential well is significant, we have

$$\frac{\mathrm{d}B_I}{\mathrm{d}\eta_1} \simeq \left(\frac{\hat{s}}{f}\right)^2 A_I \simeq \left(\frac{\hat{s}}{f}\right)^2 A_O. \tag{31}$$

In (31), we have made the constant-A approximation valid for the internal region. Matching $(B/A)_{\mathrm{O}}$ and $(B/A)_{\mathrm{I}}$, we derive the following TAE dispersion relation:

$$(-\Gamma_+/\Gamma_-)^{1/2} = \int_0^\infty \mathrm{d}\eta(\hat{s}/f)^2 = \hat{s}\pi/4 \equiv \delta T_{\mathrm{f}}. \tag{32}$$

(32) shows that for $\delta T_{\mathrm{f}} > 0$, a TAE exists and $\Omega_{\mathrm{L}}^2 < \Omega_{\mathrm{TAE}}^2 < \Omega_{\mathrm{U}}^2$.

Note that, while the external solutions are given by the singular continuum structures in the large-η (inertial) domain, δT_{f} is determined by the 'defect' structures in the $\eta \sim \mathcal{O}(1)$ ideal MHD domain. We can thus expect MHD equilibrium variations (e.g. pressure and/or current profiles) will lead to variations in δT_{f} and, thereby, TAE variations.

In the Earth's magnetosphere, magnetic pulsations with amplitudes $|\delta B/B| \sim \mathcal{O}(5-10)\%$ in the radial as well as the compressional components and high azimuthal mode numbers, $m \sim \mathcal{O}(10 - 10^2)$, have often been observed during periods of geomagnetically disturbed 'storms' [29]. Since, typically, $\beta \sim \mathcal{O}(10^{-1} - 1)$, Alfvén-ballooning modes have been proposed as the possible generating mechanism [30, 31]. Here, $\beta = \beta_{\mathrm{e}}+\beta_{\mathrm{i}}$. Noting that $m \gg 1$ and following the ballooning-mode formalism, one can also demonstrate and determine the threshold conditions for the existence of discrete radially bound eigenstates [32]. Here, the radially local $\beta \sim \mathcal{O}(1)$ plasma pressure contributes as the needed 'defects'.

4. Kinetic excitations of SAWs

That SAWs could be collectively excited by fusion-produced energetic particles such as alpha particles has long been recognized as an important issue in burning plasmas [33, 34]. My own interest in this issue was stimulated by the experimental observations of the 'fishbone' instability [35]. One important experimental insight is that the mode frequency tracks the toroidal precessional (due to magnetic gradient and curvature drifts) frequency of the energetic beam ions. That is, the mode frequency is determined by the energetic ions, not by any normal-mode oscillations of the background MHD plasmas. In a conceptually useful analogy, we may view it as the 'beam' mode in the 'beam-plasma' interacting system. It is then clear that we have to treat the energetic-ion dynamics on an equal footing with the background MHD dynamics; i.e. non-perturbatively. In other words, theoretical approach must include simultaneously both the background MHD modes and the energetic-particle modes (EPMs) [36–38]. The other insight is that since the 'fishbone' mode has a real frequency, we should expect finite Alfvén resonance absorption when the frequency matches the local SAW continuum frequency (hence, the often-used terminology, continuum damping [39–41]). This absorption should then present a threshold for the instability drive to overcome; as, indeed, observed experimentally [35].

Theoretically, due to its diluted density, we can expect, from ideal MHD considerations, that the energetic particles contribute to the SAW dynamics via pressure perturbations coupled with the finite curvature of the magnetic field lines. The difficulty is that wave–particle resonance enters into the compressional component of the pressure perturbations and must be calculated kinetically. Fortunately, by the 1980s, gyrokinetic theories including fully electromagnetic as well as nonlinear effects have advanced to a mature stage [42–44], making the calculations straightforward, if not tedious sometimes.

Let us consider high-n SAW modes to illustrate the physics of kinetic excitations. Adopting simple assumptions such as small Larmor radii, diluted energetic-particle density, $\beta < 1$, etc, the governing wave equation is then modified from (22) to the following form [12, 28, 37]

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\eta^2} + V_{\mathrm{k}}(\eta)\right]\hat{\Psi}(\eta) = 0, \qquad -\infty < \eta < \infty; \tag{33}$$

where

$$V_{\rm k}(\eta) = V_{\rm f}(\eta) + V_{\rm kc}(\eta), \tag{34}$$

$$V_{\rm f}(\eta) = \hat{\Omega}^2(1 + 2\varepsilon_0\cos\eta) + \frac{\alpha\cos\eta}{p} - \frac{(\hat{s} - \alpha\cos\eta)^2}{p^2}, \tag{35}$$

$$V_{\rm kc}(\eta)\hat{\Psi}(\eta) = -\frac{q^2R^2 4\pi\omega}{c^2k_\chi^2 p^{1/2}}\sum_j e_j\langle\omega_{\rm d}\delta G\rangle_j, \tag{36}$$

$$p(\eta) = 1 + (\hat{s}\eta - \alpha\sin\eta)^2, \tag{37}$$

$$\omega_{{\rm d}j}(\eta) = [k_\chi(v_\perp^2 + v_\parallel^2)/(\Omega_j R)][\cos\eta + (\hat{s}\eta - \alpha\sin\eta)\sin\eta], \tag{38}$$

and δG_j satisfies the linear gyrokinetic equation [30],

$$\left[\frac{v_\parallel}{qR}\frac{\partial}{\partial\eta} - {\rm i}(\omega - \omega_{\rm d})\right]\delta G_j = {\rm i}\left(\frac{e}{m}\right)_j QF_{0j}\frac{1}{p^{1/2}}\left(\frac{\omega_{\rm d}}{\omega}\right)_j\hat{\Psi}. \tag{39}$$

Here, we have introduced the $(\hat{s}, \alpha)$ equilibrium model with $\alpha = -q^2R^2\beta'$ [25], $\hat{\Omega}^2 = \omega(\omega - \omega_{*{\rm pi}})/\omega_{\rm A}^2$, $\omega_{*{\rm pi}}$ is the thermal ion diamagnetic drift frequency, $\langle\cdots\rangle \equiv \int {\rm d}^3v\ (\cdots)$, $\langle\omega_{\rm d}\delta G\rangle_j$ represents the coupling of compressional pressure perturbations of the jth species; where j = thermal electron (e), thermal ion (i) and energetic particle (E). Note that δG_j, as given by (39), contains wave–particle resonances due to the $\boldsymbol{v}_{{\rm d}B} \times \boldsymbol{B}_\perp$ force along $\boldsymbol{B}$. Here, $\boldsymbol{v}_{\rm dB}$ is the magnetic curvature and gradient drift. $\delta E_\parallel$ and $\mu\partial\delta B_\parallel/\partial l$ forces are ignored here due to, respectively, the $k_\perp^2\rho_{\rm i}^2 < 1$ and $\beta < 1$ assumptions. $QF_{0j} = [\omega\partial/\partial\mathcal{E} + (\boldsymbol{k}\times\boldsymbol{b}/\Omega_j)\cdot\nabla]F_{0j}$ with $\boldsymbol{k} = k_\chi[\hat{\boldsymbol{\chi}} + (\hat{s}\eta - \alpha\sin\eta)\hat{\boldsymbol{r}}]$ contains wave damping/growth due to velocity-space and configuration-space gradients of F_{0j}. (33) may be regarded as the generalized SAW vorticity equation.

Again, assuming $\varepsilon_0, \hat{s} < 1$, we may solve the above set of differential-integral equations via two-scale-length expansions and asymptotic matching of the solutions in the $|\eta| \gg 1$ singular 'inertial' region and the $\eta \sim \mathcal{O}(1)$ regular 'ideal MHD' region. Note that, for $|\eta| \gg 1$, the compressibility of thermal ions remains finite; while that of energetic ions is generally ignorable due to the finite-drift-orbit and finite-Larmor-radius (not shown explicitly here) suppressions.

For TAE, $\omega \simeq \omega_{\rm A}/2 \gg v_\parallel/qR$ of thermal ions, the thermal-particle contributions to $V_{\rm kc}$ can be ignored. Asymptotic matching analysis then yields the following TAE dispersion relation including the energetic-ion kinetic drive [37]

$${\rm i}\Lambda_{\rm T}(\Omega) = (-\Gamma_+/\Gamma_-)^{1/2} = \delta\hat{T}_{\rm f}(s, \alpha) + \delta T_{\rm k}(\Omega); \tag{40}$$

where $\delta\hat{T}_{\rm f} = (\hat{s}\pi/4)(1 - \alpha/\hat{s}^2)$ for $\hat{s} < 1$, and $\delta T_{\rm k}$ is the energetic-particle contribution (as an 'active defect') in the ideal region given by, for the case of untrapped circulating particles [12],

$$\delta T_{{\rm k},u} = \frac{\pi^2}{4\hat{s}}\left(\frac{e^2}{mc^2}\right)_{\rm E}\frac{q^2R^2}{k_\chi^2}\left\langle\frac{\Omega_{\rm d}^2 QF_0}{\Delta_{\rm d}(1 + \Delta_{\rm d})^{3/2}}\left[\frac{\omega}{\omega_{\rm t}^2/4 - \omega^2} + \frac{\omega}{9\omega_{\rm t}^2/4 - \omega^2}\right]\right\rangle_{\rm E}; \tag{41}$$

where $\Omega_{\rm d} = k_\chi(v_\perp^2/2 + v_\parallel^2)/(\Omega_{\rm c}R)$, $\Delta_{\rm d} = (k_\chi^2/4)(\rho_{\rm L}^2 + \rho_{\rm d}^2/2)$, $\rho_{\rm L}$ being the Larmor radius, $\rho_{\rm d} = \Omega_{\rm d}/(k_\chi\omega_{\rm t})$ and $\omega_{\rm t} = v_\parallel/qR$ being the transit frequency. The wave–particle resonances in (41) correspond to $p = 0$ and $p = 1$ choices of the following general resonance condition:

$$\omega - k_\parallel v_\parallel - p\omega_{\rm t} = 0; \qquad p = \text{integers},$$

and $k_\parallel = 1/2qR$. Similar expressions can also be derived for the trapped particles [12]. Note that in (40), the singular 'inertial' term, ${\rm i}\Lambda_{\rm T}(\Omega)$, the fluid term, $\delta\hat{T}_{\rm f}$, and the energetic-particle term $\delta T_{\rm k}(\Omega)$ are, in principle, of the same order. That is, $\delta T_{\rm k}$'s contribution is non-perturbative.

(40) indicated two types of unstable eigenmodes. One is the (toroidal Alfvén) gap eigenmode for $\mathrm{Re}(\Lambda_T^2) < 0$; i.e. TAE with $\Omega_L^2 < \Omega^2 < \Omega_U^2$; whose existence requires that $\delta\hat{T}_f + \mathrm{Re}\delta T_k(\Omega) > 0$; otherwise, Ω^2 merges into the SAW continuum. Instability, meanwhile, will set in if the energetic-particle drive $\mathrm{Im}(\delta T_k)$ overcomes thermal-particle damping, $\mathrm{Re}(\Lambda_T)$, which could be incorporated into $\Gamma_\pm$ by keeping finite wave–particle interactions in the thermal-particle pressure compression [28]. The other type of unstable eigenmodes corresponds to $\mathrm{Re}(\Lambda_T^2) > 0$; i.e. Ω^2 resides outside the gap and in the continuum. Its existence then requires $\mathrm{Im}(\delta T_k) \geqslant \mathrm{Re}(\Lambda_T)$ or that the instability drive exceeds the Alfvén continuum damping. Furthermore, near the instability threshold, we have

$$\mathrm{Re}[\delta T_k(\Omega)] \simeq -\delta\hat{T}_f. \tag{42}$$

Since $\delta\hat{T}_f$ is independent of Ω and $\delta T_k(\Omega)$ contains only energetic-particle's characteristic dynamic frequency, (42) demonstrates that the mode frequency of the unstable 'continuum' eigenmode must track the energetic-particle characteristic frequencies; such as transit frequency ω_{tE} in (41), or the bounce and/or toroidal precessional frequencies, ω_{bE} and $\overline{\omega}_{dE}$, in the case of trapped particles. Hence, this type of eigenmodes is termed as EPMs [37].

At further lower frequency, SAW in tokamaks has its end point (i.e. accumulation point) at $\Omega^2 = 0$. Here, the physics and, hence, the analysis become more complicated and, perhaps, more interesting. Complication arises because at this low end, the SAW frequency becomes comparable to the thermal-particle dynamic frequencies; e.g. the diamagnetic drift, transit, bounce and toroidal precessional frequencies. We can, however, still perform a two-scale-length asymptotic matching analysis and obtain a dispersion relation of the following form

$$\mathrm{i}\Lambda_L(\Omega) = \delta W_f + \delta W_k(\Omega). \tag{43}$$

Again, Λ_L, δW_f and δW_k correspond, respectively, to contributions from the 'singular' inertial region, ideal MHD fluid and the energetic-particle pressure compression. Ignoring the thermal-particle pressure compression and assuming only trapped energetic ions, (43) becomes

$$\mathrm{i}\hat{\Omega} = \delta W_f + 2(\pi q/B)_E^2 m_E \langle (v_\perp^2/2)^2 Q F_0/(\bar{\omega}_d - \omega)\rangle_E/\hat{s}. \tag{44}$$

Here, $\bar{\omega}_d$ is the toroidal precessional frequency. (44) is first derived in its present form for the $n = 1$ internal-kink 'fishbone' instability [36].

Thus, both (40) and (43) may be regarded as generalized 'fishbone' dispersion relations. Similar to (40), (43) demonstrates the existence of two types of discrete unstable eigenmodes; one (kinetic thermal ion) 'gap' mode given by $\mathrm{Re}(\Lambda_L^2) < 0$, and the other being the EPM inside the SAW continuum given by $\mathrm{Re}(\Lambda_L^2) > 0$. In this respect, the 'fishbone' mode with its frequency tracking the energetic-particle toroidal precessional frequency is the experimentally observed first EPM. Furthermore, we note that, for typical tokamak parameters, thermal ion compression effects could become significant in the inertial layer and the corresponding expression of Λ_L has been derived in [45, 46]. More detailed discussions on the generalized fishbone dispersion and roles of non-resonant and resonant energetic particles are found in [46].

Advances made in the theoretical studies of kinetic excitations of SAW in tokamak fusion plasmas have stimulated systematic gyrokinetic studies of geomagnetic pulsations internally generated by energetic ions injected into the inner magnetosphere during geomagnetic storms [29]. Distinctive from tokamak fusion plasmas, these injected energetic ions have, typically, $\beta \sim \mathcal{O}(1)$, $T_\perp > T_\parallel$, and, of course, are magnetically trapped. We, thus, have the interesting possibilities of collective excitations of instabilities known in mirror fusion research; e.g. mirror mode [47, 48] and trapped-particle compressional mode [49–51]; both of which involve significant compressional magnetic perturbations. For general equilibria and parameters, the mirror-compressional mode is coupled with the Alfvén-ballooning mode [30]. Both modes have been studied in detail only in the various decoupled limits. In the case of the Alfvén-ballooning mode, for example, the trapped-particle compressional stabilization renders the most significant kinetic excitation to be the precessional drift-bounce resonance; i.e. $p = -1$ in the following wave-trapped particle resonance condition

$$\omega - \bar{\omega}_{\mathrm{d}} - p\omega_{\mathrm{b}} = 0; \qquad p \text{ integers.}$$

Precessional drift-bounce resonance has also been suggested for the energetic-particle driven Alfvén instabilities in NSTX [52]. Comprehensive studies involving coupled mirror-compressional and Alfvén-ballooning modes in realistic magnetospheric plasma geometries, however, remain to be done.

5. Nonlinear physics of Alfvén waves

Nonlinear physics associated with SAWs covers a broad scope of actively ongoing research efforts. They range from examining the nonlinear dynamics of charged particles due to SAW wave(s) [53–55] to studying the spectral cascading due to nonlinear mode-mode coupling processes. I will, therefore, have to consider a narrower scope of topics due to both my own limited research exposure and the need to achieve some coherence in the following discussions.

It is well known that, in an incompressible ideal MHD plasma confined by a uniform $\boldsymbol{B}_0$, a finite-amplitude SAW corresponds to a self-consistent nonlinear state [56]. This pure Alfvénic state is characterized by the following so-called Walén relation [57]:

$$\frac{\delta \boldsymbol{u}}{V_{\mathrm{A}}} = \pm \frac{\delta \boldsymbol{B}}{B_0}, \tag{45}$$

and satisfies the SAW dispersion relation $\omega^2 = k_\parallel^2 V_{\mathrm{A}}^2$. In terms of the nonlinear ponderomotive force, the pure Alfvénic state also corresponds to exact cancellation between the Reynolds stress, $\rho_{\mathrm{m}}(\delta\boldsymbol{u}\cdot\nabla)\delta\boldsymbol{u}$, and the Maxwell stress, $(\delta\boldsymbol{B}\cdot\nabla)\delta\boldsymbol{B}/4\pi$, from the $(\delta\boldsymbol{J}\times\delta\boldsymbol{B})/c$ force in the SAW vorticity equation.

For low-β fusion plasmas, since $\delta\boldsymbol{u} \simeq \delta\boldsymbol{u}_\perp$ for SAWs and $\nabla_\perp\cdot\delta\boldsymbol{u} \simeq 0$ if $\boldsymbol{B}_0$ is uniform, the ponderomotive force, on average, balances out in the perpendicular to $\boldsymbol{B}$ direction. Ideal SAWs are, thus, inefficient in generating perpendicular flows. Note that the purely Alfvénic state described by the Walén relation implies two necessary conditions; $\delta E_\parallel = 0$ and $\omega^2 = k_\parallel^2 V_{\mathrm{A}}^2$. Thus, effects that break either of the two conditions will lead to significant perpendicular ponderomotive force and, hence, zonal flows [58, 59]. In terms of discussions presented in the preceding sections, one would expect that KAW and EPM would be more effective than TAE in direct generation of perpendicular flows.

While, due to the strong $\boldsymbol{B}_0$, incompressiblity remains generally valid in the perpendicular direction, parallel compressibility becomes effective at lower frequencies, such as the slow-sound frequency. Here, the cancellation of parallel ponderomotive force does not occur and the $(\delta\boldsymbol{J}_\perp\times\delta\boldsymbol{B}_\perp)\cdot\boldsymbol{b}$ force dominates. It has long been recognized that this parallel ponderomotive force can couple with the compressible dynamics of slow sound waves and lead to various parametric scattering and decay processes [11, 60]. To be more specific, for ideal SAW, we have

$$\boldsymbol{F}_{\mathrm{p}\parallel} = -(1/8\pi n_0)\boldsymbol{b}\cdot\nabla\left[(\delta B_\perp)^2\right]_{\mathrm{s}} \equiv -e\boldsymbol{b}\cdot\nabla\Phi_{\mathrm{ps}}. \tag{46}$$

Here, $[A]_{\mathrm{s}}$ denotes the slow sound-wave temporal component of A, and

$$\delta\Phi_{\mathrm{ps}} = (1/8\pi n_0 e)[(\delta B_\perp)^2 - \overline{(\delta B_\perp)^2}], \tag{47}$$

where $\bar{A}$ denotes the line average of A. $\boldsymbol{F}_{\mathrm{p}\parallel}$ will excite ion density perturbations along $\boldsymbol{B}$ and, in turn, a self-consistent electrostatic potential variation

$$\mathcal{D}_{\mathrm{s}}\delta\phi_{\mathrm{s}} = -\chi_{\mathrm{is}}\delta\Phi_{\mathrm{ps}}; \tag{48}$$

where $\mathcal{D}_s \simeq \chi_{es} + \chi_{is}$, with χ_{js} being the jth species' susceptibility operator in the slow-sound frequency range. In (48), we have taken the quasi-neutrality, $\delta n_{es} \simeq \delta n_{is}$, approximation. The slow-sound density perturbation is then given by

$$\frac{\delta n_{is}}{n_0} \simeq -\chi_{es}(1 - \mathcal{D}_s^{-1}\chi_{es})\Phi_{ps}, \tag{49}$$

and δn_{is} couples into the SAW dynamics via the ion polarization current (inertial-layer) physics.

For TAEs, $\delta B_\perp$ has $|k_\parallel| \sim 1/2qR$ and, hence, $\Phi_{ps}(\delta n_s)$ has $|k_{\parallel s}| \sim 1/qR \sim 2|k_\parallel|$. Now, if the two interacting TAEs have different n numbers, n_1 and n_2, such that $|\Delta\omega| = |\omega_1 - \omega_2| \sim \mathcal{O}(\varepsilon_0 V_A/2qR) \sim |k_{\parallel s} v_{ti}|$, higher-frequency TAEs can then be back-scattered into lower-frequency TAEs with enhanced continuum damping via nonlinear ion Landau damping due to finite $\mathrm{Im}(\chi_{is})$ [61]. Similar processes may apply in the saturation of Alfvén-ballooning modes in the Earth's magnetosphere [62].

On the other hand, for a single-n TAE, Φ_{ps} and, hence, δn_s and $\delta\phi_s$, will have radially varying ($n = 0, m = \pm 1$) poloidal structures; which can lead to radially local narrowing in the toroidal frequency gap and, consequently, stabilization via enhanced continuum damping [63]. Similarly, noting that, in tokamaks, perpendicular incompressibility of SAW is broken by the magnetic field line curvature; $\boldsymbol{\nabla}_\perp \cdot \delta \boldsymbol{u}_\perp \simeq -2\boldsymbol{\kappa}_c \cdot \delta \boldsymbol{u}_\perp$ with $\boldsymbol{\kappa}_c = \boldsymbol{b} \cdot \boldsymbol{\nabla b}$, it has been demonstrated that TAE can then nonlinearly generate ($n = 0, m = \pm 1$) radially local magnetic perturbations via

$$\partial \delta \boldsymbol{B}_\perp / \partial t = (\boldsymbol{B}_0 \cdot \boldsymbol{\nabla})\delta \boldsymbol{u}_\perp - \delta \boldsymbol{B}_\perp(\boldsymbol{\nabla} \cdot \delta \boldsymbol{u}_\perp); \tag{50}$$

which, in turn, modifies the magnetic surface; leading, again, to the narrowing of the toroidal Alfvén frequency gap and saturation of the TAE instabilities [64].

The above discussion suggests several interesting and yet theoretically unexplored physics issues. For example, it is well known that, in tokamaks, radially varying zonal perturbations carry ($n = 0, m = \pm 1$) density perturbations; there is, thus, the interesting possibility of nonlinear excitations of zonal perturbations via the parallel ponderomotive force of SAW such as TAE. Furthermore, as noted in [11], the parallel ponderomotive force due to KAW is typically $\mathcal{O}(|k_\perp \rho_i|\Omega_i/\omega)$ larger than that given by (46) derived in the ideal MHD limit. Physically, this enhancement comes about since $\delta \boldsymbol{J}_\perp$ in the ideal MHD limit corresponds to the ion polarization current; while, for KAW, $\delta \boldsymbol{J}_\perp$ is due to the difference in the $(\delta \boldsymbol{E}_\perp \times \boldsymbol{B})/B^2$ drift between electrons and ions subject to the finite-Larmor-radius effects. That the nonlinear mode-coupling process can be significantly enhanced indicates the importance of keeping the relevant microscopic thermal ion physics in the investigation of Alfén wave dynamics. Other interesting nonlinear Alfvén wave physics issues relevant to burning tokamak plasmas such as coupled nonlinear physics of energetic particles and multiple AEs, nonlinear physics of EPM, and long-time-scale coupling between meso-scale SAW instabilities and micro-scale thermal-particle turbulence have been explored in [65, 66].

In magnetospheric space plasmas, theoretical studies of nonlinear Alfvén wave physics have been rather sparse. One obvious reason is that the solar-terrestrial environment is more dynamic and less controllable. The other reason is that, as noted earlier, the magnetospheric plasma consists, typically, of high-β, anisotropic, magnetically trapped particles, and the mirror-compressional mode is usually coupled with the Alfvén-ballooning mode in realistic dipole-like magnetic field geometries. Theoretical studies including simulations are hence often limited to various decoupled simplifying limits [62, 67, 68]. Hopefully, the rapid advances in the gyrokinetic simulations [69] along with theoretical understandings of nonlinear physics in fusion plasmas can stimulate more comprehensive investigations in the dynamics of magnetospheric space plasmas.

6. Concluding remarks

In this lecture, I, hopefully, have demonstrated that, in plasmas confined by realistic magnetic field geometries, the anisotropic and nearly incompressible SAWs exhibit rich and interesting physics.

First, it is shown that, due to perpendicular to ***B*** (radial) non-uniformities, SAW possesses a continuous spectrum and could experience continuum damping via resonant absorption. Consequently, SAW structures become singular time asymptotically and can be mode converted to KAWs with microscopic perpendicular (radial) wavelengths. One can view this mode conversion as a cross-scale coupling among radial wavelengths. Since the microscopic KAW carries finite parallel electric field, it can lead to acceleration, heating and cross ***B*** transport of charged particles. It is then demonstrated that, while periodic 'lattice' non-uniformities along ***B*** produce gaps in the SAW continuum, localized 'defects' can introduce discrete Alfvén eigenmodes (AEs) within the gaps. Since these AEs experience negligible continuum damping, they can be collectively excited via wave–particle resonances. Such wave–particle resonances are shown to enter into the SAW dynamics via finite ***B*** curvature coupling to the plasma pressure compression. In general, the instability driving energetic particles must be treated non-perturbatively and corresponding generalized fishbone dispersion relations can be derived. Two types of discrete eigenmodes can then exist. One is the (gap) AE residing inside the SAW continuum gap, and the other is the (continuum) EPM which resides inside the continuum and requires stronger drive to overcome the finite continuum damping.

As to the nonlinear SAW physics, while noting the crucial roles of nonlinear phase-space dynamics of charged particles due to the presence of wave–particle resonances, the focus here is on the nonlinear mode-coupling processes. In particular, a conceptual framework in terms of ponderomotive forces is adopted here. The pure Alfvénic state described by the Walén relation in an incompressible ideal MHD plasma confined by a uniform ***B*** then corresponds to exact cancellation of the ponderomotive forces in the SAW vorticity equation. Deviations from the pure Alfvénic state due to, e.g. finite $\delta E_{\|}$, plasma compressibility and non-perturbative energetic particles, then result in significant nonlinear mode-coupling effects. In this respect, I remark on the roles of parallel ponderomotive force and parallel plasma compressibility at the slow-sound frequency range, the perpendicular compressibility due to ***B*** geometry and on the enhanced parallel ponderomotive force in the presence of microscopic structures.

Ultimately, I wish that my own humble research on Alfvén wave physics in space and fusion plasmas demonstrates the positive and fruitful feedback and cross-fertilization between these two important subjects of plasma physics research. Furthermore, as universal to every area of plasma physics research, advances in Alfvén wave physics research have been benefited from successful feedback and close collaborations between experiments/observations, theories and simulations.

Finally, as noted in this lecture, there remain many exciting and challenging Alfvén wave physics issues; especially in the nonlinear areas. Some of them I have explored in more depth due to my own limited understanding. All these considerations indicate that one needs to properly take into account realistic plasma geometries and conditions. This is because these effects; e.g. non-uniformities and magnetic field line curvature, play such crucial roles in determining the SAW dynamics and structures. Therefore, serious and coordinated efforts in simulations, theory, and experiment/observation are, in my view, necessary to further advance the Alfvén wave physics research which is both intellectually exciting and practically important.

Looking back, my 'journey' of Alfvén wave physics research may be summarized by the following sayings of Kongtze (Confucius), in the very beginning of his Analects (my own loose translation):

'Learning and studying so very often, what a pleasure!
Having friends from far away places, what a happiness!'

-Kongtze

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