Nonlinear MHD effects on TAE evolution and TAE bursts

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Outline

● MEGA code

● Nonlinear MHD effects on TAE evolution

● Simulation of TAE bursts using time dependent $f_0$
MHD Equations with EP effects

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) + \nu_n \Delta \rho
\]

\[
\rho \frac{\partial}{\partial t} \mathbf{v} = -\rho \omega \times \mathbf{v} - \rho \nabla \left( \frac{\mathbf{v}^2}{2} \right) - \nabla p + (\mathbf{j} - \mathbf{j}_n) \times \mathbf{B} + \frac{4}{3} \nabla \left[ \nu \rho (\nabla \cdot \mathbf{v}) \right] - \nabla \times \left[ \nu \rho \omega \right]
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]

\[
\frac{\partial p}{\partial t} = -\nabla \cdot (p \mathbf{v}) - (\gamma - 1) p \nabla \cdot \mathbf{v} + (\gamma - 1) \left[ \nu \rho \omega^2 + \frac{4}{3} \nu \rho (\nabla \cdot \mathbf{v})^2 + \eta \mathbf{j} \cdot (\mathbf{j} - \mathbf{j}_{eq}) \right] + \nu_n \Delta p
\]

\[
\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta (\mathbf{j} - \mathbf{j}_{eq})
\]

\[
\mathbf{j} = \frac{1}{\mu_0} \nabla \times \mathbf{B}
\]

\[
\omega = \nabla \times \mathbf{v}
\]
Energetic particle current density

Energetic ion current density without ExB drift:

\[ j'_h = \int q_h (v^*_\parallel + v_B) f d^3v - \nabla \times \int \mu f b d^3v \]

parallel + curvature drift + grad-B drift \hspace{1cm} \text{magnetization current}

The energetic ion pressures are calculated using the \( \delta f \) PIC method.

\[ P_{h,\parallel}(x) = P_{h,\parallel 0}(x) + \sum_{i}^{N} m_i v^2_{\parallel i} w_i S(x - x_i), \]

\[ P_{h,\perp}(x) = P_{h,\perp 0}(x) \frac{B(x)}{B_0(x)} + B(x) \sum_{i}^{N} \mu_i w_i S(x - x_i). \]
Simulation Study of Nonlinear Magnetohydrodynamic Effects on Alfvén Eigenmode Evolution and Zonal Flow Generation
Outline

- Introduction - simulation of TAE bursts
- Numerical investigation of nonlinear MHD effects on TAE (n=4) evolution
  - Comparison of linear and NL MHD simulations
    - MHD nonlinearity suppresses the TAE saturation level
  - Suppression mechanism of TAE saturation level
  - Spatial profiles and evolution of nonlinearily-generated n=0 mode
  - Excitation of geodesic acoustic mode (GAM)
- Summary
Reduced Simulation of Alfvén Eigenmode Bursts

[Todo, Berk, Breizman, PoP 10, 2888 (2003)]

- Nonlinear simulation in an open system:

  NBI, collisions, losses

- The experimental results were reproduced quantitatively.

Store of energetic ions

Destabilization of AEs

Transport and loss of energetic ions

Stabilization of AEs

Time evolution of energetic-ion density profile.
consistency and inconsistency

- consistent with the experiment:
  - synchronization of multiple TAEs
  - drop in stored beam energy at each burst
  - burst time interval
- inconsistent in saturation amplitude
  - simulation $\delta B/B \sim 10^{-2}$
  - inferred from the experiment $\delta B/B \sim 10^{-3}$
Numerical investigation of nonlinear (NL) MHD effects on a TAE (n=4) evolution
Initial Plasma Profile

\[ \beta_h = \beta_{h0} \exp[-(r / 0.4a)^2] \]
\[ q = 1 + 2(r / a)^2 \]
\[ a\Omega_h / v_A = 16 \]
\[ R_0 / a = 3.2 \]

\[ \beta_{h0} = 2\% \]
\[ f(v, \mu, \sigma, P_\psi) \approx g(v) h(\psi) \]
\[ g(v) = \frac{1}{|v|^3 + v_c^3} \left[ 2 \left( 1 - \tanh \left( \frac{|v - v_b|}{\Delta v} \right) \right) \right] \]
\[ v_b = 1.2v_A, \; v_c = 0.5v_A, \; \Delta v = 0.1v_b \]
The main harmonics are $m=5$ and 6.
Comparison between linear and NL MHD runs (\(j_h'\) is restricted to n=4)

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho_{eq} v) + \nu_n \Delta (\rho - \rho_{eq})
\]

\[
\rho_{eq} \frac{\partial}{\partial t} v = -\nabla p + (j_{eq} - j_{h_{eq}}') \times B + (\delta j - \delta j_{h}') \times B_{eq}
\]

\[
+ \frac{4}{3} \nabla (\nu \rho_{eq} \nabla \cdot v) - \nabla \times (\nu \rho_{eq} \omega)
\]

\[
\frac{\partial B}{\partial t} = -\nabla \times E
\]

\[
\frac{\partial p}{\partial t} = -\nabla \cdot (p_{eq} v) - (\gamma - 1) p_{eq} \nabla \cdot v + \nu_n \Delta (p - p_{eq})
\]

\[
+ \eta \delta j \cdot j_{eq}
\]

\[
E = -v \times B_{eq} + \eta (j - j_{eq})
\]

\[
j = \frac{1}{\mu_0} \nabla \times B
\]

\[
\omega = \nabla \times v
\]

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho v) + \nu_n \Delta (\rho - \rho_{eq})
\]

\[
\rho \frac{\partial}{\partial t} v = -\rho \omega \times v - \rho \nabla \left( \frac{v^2}{2} \right) - \nabla p + (j - j_{h}) \times B
\]

\[
+ \frac{4}{3} \nabla (\nu \rho \nabla \cdot v) - \nabla \times (\nu \rho \omega)
\]

\[
\frac{\partial B}{\partial t} = -\nabla \times E
\]

\[
\frac{\partial p}{\partial t} = -\nabla \cdot (p v) - (\gamma - 1) p \nabla \cdot v + \nu_n \Delta (p - p_{eq})
\]

\[
+ (\gamma - 1) [\nu \omega^2 + \frac{4}{3} \nu (\nabla \cdot v)^2 + \eta \cdot (j - j_{eq})]
\]

\[
E = -v \times B + \eta (j - j_{eq})
\]

\[
j = \frac{1}{\mu_0} \nabla \times B
\]

\[
\omega = \nabla \times v
\]

The viscosity and resistivity are \(\nu = \nu_n = 10^{-6} v_A R_0\) and \(\eta = 10^{-6} \mu_0 v_A R_0\).

The numbers of grid points are (128, 64, 128) for (R, \(\phi\), z).

The number of marker particles is \(5.2 \times 10^5\). 0\(\leq \phi \leq \pi/2\) for the n=4 mode.
Comparison of linear and NL runs

\[ \beta_{h0} = 1.5\% \]
- Sat. Level (linear) ~ 3x10^{-3}
- Sat. Level (NL) ~ 3x10^{-3}

\[ \beta_{h0} = 2.0\% \]
- Sat. Level (linear) ~ 1.6x10^{-2}
- Sat. Level (NL) ~ 8x10^{-3}

The saturation level is reduced to half in the nonlinear run.
Suppression mechanism of TAE saturation level (1/2)

Energy of each toroidal mode number $n$ ($n=0, 4, 8, 12, 16$)

$$E_n = \frac{1}{2} \int \left( \rho_{n=0} v_n^2 + (B - B_{eq})_n^2 \right) dV$$

Energy dissipation of each toroidal mode number $n$ ($n=0, 4, 8, 12, 16$)

$$D_n = \int \left[ \nu \rho_{n=0} \omega_n^2 + \frac{4}{3} \nu \rho_{n=0} (\nabla \cdot v_n)^2 + n j_n \cdot (j - j_{eq})_n \right] dV$$

Damping rate of each toroidal mode number $n$ ($n=0, 4, 8, 12, 16$)

$$\gamma_{d,n} = \frac{D_n}{2E_4}$$

Total damping rate of all the toroidal mode numbers ($n=0, 4, 8, 12, 16$)

$$\gamma_{d, ALL} = \sum_n \frac{D_n}{2E_4}$$
Suppression mechanism of TAE saturation level (2/2)

The total damping rate ($\gamma_{d\,\text{ALL}}$) is greater than the damping rate in the linearized MHD simulation ($\gamma_{d\,\text{lin}}$).

$\beta_{h0}=1.7\%$
Sat. Level (linear) $\sim 1.2 \times 10^{-2}$
Sat. Level (NL) $\sim 6 \times 10^{-3}$
Schematic Diagram of Energy Transfer

Energetic Particles

Drive

n=4 TAE

Dissipation

Linearized MHD

n=0 and higher-n modes

NL coupling

Thermal Energy

Dissipation

NL coupled modes

Thermal Energy
Effects of weak dissipation

The nonlinear MHD effects reduce the saturation level also for weak dissipation.

\[ \beta_{h0} = 1.7\% \]

The viscosity and resistivity are reduced to \( 1/4 \),

\[ \nu = \nu_n = 2.5 \times 10^{-7} \nu_A R_0 \quad \text{and} \quad \eta = 2.5 \times 10^{-7} \mu_0 \nu_A R_0 . \]

The nonlinear MHD effects reduce the saturation level also for weak dissipation.
Excitation of GAM

After the saturation of the TAE instability, a geodesic acoustic mode is excited.
Summary of NL MHD effects on a TAE instability

- Linear and nonlinear simulation runs of a n=4 TAE evolution were compared. The saturation level is reduced by the nonlinear MHD effects.

- The total energy dissipation is significantly increased by the nonlinearly generated modes. The increase in the total energy dissipation reduces the TAE saturation level.

- The zonal flow is generated during the linearly growing phase of the TAE instability. The geodesic acoustic mode (GAM) is excited after the saturation of the instability. The GAM is not directly excited by the energetic particles but excited through MHD nonlinearity.
Simulation of Alfvén eigenmode bursts with nonlinear MHD effects
Simulation with source, loss, collisions and NL MHD

- Time dependent $f_0$ is implemented in MEGA
- Particle loss (at $r/a=0.8$ for the present runs)
  - For marker particles to excurse outside the loss boundary and return back to the inside
  - Phase space inside the loss boundary is well filled with the marker particles
  - Particle weight is set to be 0 during $r/a>0.8$
  - If the simulation box is extended to include the vacuum region, the loss boundary can be set at more realistic location
δf method with time-dependent $f_0$ (1/2)

\[
\frac{\partial}{\partial t} f + \{H, f\} - \nu \frac{\partial}{\partial v} \left[ \left( v^3 + v_c^3 \right) f \right] = S(v)
\]

When $f_0$ satisfies

\[
\frac{\partial}{\partial t} f_0 + \{H_0, f_0\} - \nu \frac{\partial}{\partial v} \left[ \left( v^3 + v_c^3 \right) f_0 \right] = S(v),
\]

the evolution of $\delta f$ is given by

\[
\frac{\partial}{\partial t} \delta f + \{H_0 + H_1, \delta f\} + \{H_1, f_0\} - \nu \frac{\partial}{\partial v} \left[ \left( v^3 + v_c^3 \right) \delta f \right] = 0.
\]

With a definition $d \delta f = \frac{\partial}{\partial t} \delta f + \{H_0 + H_1, \delta f\} - \nu \left( 1 + \frac{v_c^3}{v^3} \right) \frac{\partial}{\partial v} \delta f$,

the evolution of $\delta f$ is expressed by

\[
\frac{d}{dt} \delta f + \{H_1, f_0\} - 3\nu \delta f = 0.
\]
δf method with time-dependent \( f_0 \) (2/2)

The evolution of phase space volume \( V \) that each particle occupies should be considered. Comparison of two eqs.:

\[
\frac{d}{dt}(fV) = SV
\]

and

\[
\frac{d}{dt} f - 3vf = S
\]

gives

\[
\frac{d}{dt} V = -3vV
\]

We solve the evolution of both \( δf \) and \( V \) of marker particles.
Time-dependent $f_0$

A solution of

$$\frac{\partial}{\partial t} f_0 - \nu \frac{\partial}{\partial v} \left[ \left( v^3 + v_c^3 \right) f_0 \right] = S(v) ,$$

is

$$f_0(v, t) = \frac{1}{\nu} \frac{1}{v^3 + v_c^3} \left[ \text{erf} \left( \frac{v' - v_b}{\Delta v} \right) - \text{erf} \left( \frac{v - v_b}{\Delta v} \right) \right]$$

with

$$v' = \left[ \left( v^3 + v_c^3 \right) \exp \left( 3 \nu (t + t_{inj}) \right) - v_c^3 \right]^{1/3} ,$$

$$S(v) = \frac{2}{\sqrt{\pi}} \frac{1}{v^2 \Delta v} \exp \left[ - \left( \frac{v - v_b}{\Delta v} \right)^2 \right] ,$$

$v_b$: injection or birth velocity of energetic particle

$t_{inj}$: injection starts at $t = -t_{inj} < 0$

Note: Here we have neglected $\{H_0, f_0\}$ term for parallel beam injection (zero magnetic moment) and w/o finite orbit width effect.
Physics condition of simulation

- similar to the previous reduced simulation of TAE bursts with a few exceptions
- parameters
  - $a=0.75\text{m}$, $R_0=2.4\text{m}$, $B_0=1\text{T}$, $q(r)=1.2+1.8(r/a)^2$
  - NBI power: 10MW
  - beam injection energy: 110keV (deuterium)
  - $v_b=v_A$
  - parallel injection ($v_{//}/v=-1$ or 1)
  - slowing time: 100ms
  - no pitch angle scattering

The time-dependent $f_0$ is similar to but slightly different from that in the previous slide.
Evolution of TAE amplitude and stored beam energy

- Energetic particle losses due to TAE bursts.
- Synchronization of multiple TAEs.

(Amplitude is measured at the mode peak locations.)
Comparison of NL and linear MHD runs

Linear MHD

NL MHD

NL MHD effects: reduction of saturation level, rapid damping after saturation, and suppression of beam ion loss.
Spatial profile and evolution of zonal flow

$n=0$ poloidal flow profile at $t=8.03\text{ms}$.

evolution of zonal flow at $r/a=0.28$ (top) and $r/a=0.53$ (bottom).
Summary of TAE burst simulation

- TAE bursts are successfully simulated with NL MHD effects using time-dependent $f_0$.
  - synchronization of multiple TAEs
  - beam ion loss at each burst
  - (maximum saturation level: $\delta B/B \sim 9 \times 10^{-3}$)

- NL MHD effects:
  - reduction of saturation level
  - rapid damping after saturation
  - suppression of beam ion loss